Journal of Aeronautics and Space Technologies
17(2) (2024) 1-17
Journal Homepage: jast.hho.msu.edu.tr

RESEARCH ARTICLE

Influence of Time Intervals: Comparative Analysis of Short-Arc Orbit Determination for GEO Satellites Using AER and RNG Methods

Kısa Süreli Ölçüm ile Yer Sabit Uydu Yörüngesi Belirlemede Sürenin YYM ve MM Yöntemleriyle Karşılaştırmalı Analizi

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***Received:** January 2, 2024* ***Revised:** July 2, 2024* ***Accepted:** July 11, 2024*

Abstract

Short arc orbit determination for GEO satellites is useful when there are gaps or incomplete data due to limited time or other factors. Full cycle orbit determination requires continuous data over the entire orbit cycle, which may not always be available. This research assesses the accuracy of short arc orbit determination for GEO satellites for various time intervals ranging from 6-hour- to 42-hour intervals using two data collecting techniques: azimuth elevation range (AER) and range-to-range (RNG). Iterative Least-Squares Estimation, a numerically integrated method, is then used to determine the orbit, by either calculating new bias values or pre-existing bias values. The results show that average root mean square error (RMSE) values are 0.792 km and 0.160 km for AER and RNG type observation data, respectively, using pre-existing bias values. On the other hand, using new bias calculation, the average RMSE values are 36.480 km and 6.254 km for AER and RNG. The findings indicate that the RNG method provides superior accuracy in determining the orbit for both satellites. The study recommends the use of pre-existing bias values over calculating new ones. Short arc orbit determination can provide a reasonable estimate of the satellite's orbit when a limited amount of data is available due to various reasons and can also be used to quickly assess and correct any deviations or errors in the satellite's orbit, improving overall accuracy and reliability.

Keywords: Short-arc orbit determination, Orbit, bias value, Range method, Azimuth elevation range method

Öz

Yer sabit yörüngedeki (GEO) uyduların yörüngesi, sınırlı süre, ölçüm verisi azlığı gibi durumlarda kısa ölçümlerle belirlenebilmektedir. Tam döngü yörünge belirleme, döngü boyunca sürekli veri gerektirmekte olup bu her zaman mümkün olmayabilir. Bu araştırma, 6 saat ile 42 saat arasında değişen çeşitli zaman aralıkları için GEO uyduları için kısa süreli yörünge belirleme hassasiyetini analiz etmektedir. Bu süreçte en çok kullanılan iki veri toplama tekniği olan yan-yükseliş-mesafe (YYM) ve mesafe-mesafe (MM) ölçümleri kullanılmıştır. Elde edilen veriler, Yinelemeli En Küçük Kareler yöntemi ile, bias değeri yeniden hesaplanarak veya mevcut bias değeri kullanılarak yörünge belirlemede kullanılmıştır. YYM ve MM yöntemi ile mevcut bias verileri kullanıldığında 0,792 km ve 0,160 km kök ortalama kare hatası hesaplanmıştır. Bias değeri yeniden hesaplandığında YYM ve MM için kök ortalama kare hata değerleri sırasıyla 36,480 km ve 6,254 km olarak elde edilmiştir. Bulgular, YY yönteminin her iki uydu için de yörünge belirlemede daha iyi sonuç verdiğini göstermektedir. Bu araştırma, yeni bias değerleri hesaplamak yerine mevcut bias değerlerinin kullanılmasını önermektedir. Kısa süreli ölçüm ile yörünge belirleme, sınırlı ölçüm verisi olduğu ve zamanın sınırlı olduğu durumlarda kullanılmakta ve makul sonuçlar üretmektedir ayrıca uydu yörüngesindeki herhangi bir sapmayı veya hatayı hızlı bir şekilde değerlendirme ve düzeltme imkânı sağlamaktadır.

Anahtar Kelimeler: Kısa süreli ölçümle yörünge belirleme, Yörünge, Bias, Mesafe-mesafe yöntemi, Yan yükseliş mesafe yöntemi

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1. INTRODUCTION

Orbit determination is the process of determining an object's past, present, or future position and velocity in space. Several methods can be used to determine the orbit of an object, including;

a) Analytical orbit determination: This method uses a set of equations that describe the motion of an object under the influence of gravitational forces to determine its orbit, b) Numerical orbit determination: This method uses numerical integration techniques to solve the equations of motion and determine the orbit of an object, c) Least squares orbit determination: This method uses a set of observations of an object's position and velocity to fit a mathematical model of its orbit, d) Monte Carlo orbit determination: This method uses statistical techniques to determine the orbit of an object by simulating a large number of possible orbits and selecting the one that is most likely based on the available observations [1, 2].

It is important to note that the accuracy of orbit determination results depends on the quality of the observations used, the mathematical models and algorithms employed, and the computational resources available [3-6]. The duration of the observation period can significantly affect the accuracy of geostationary orbit determination [7, 8]. However, other factors can affect the accuracy of orbit determination, such as the quality of the observations (e.g. the precision of the measurement equipment), the number of observations taken, and the presence of any external forces (such as the Earth's gravitational field) that may affect the satellite's motion [9-13]. Hence, meticulous consideration of various factors becomes crucial in determining the optimal observation period duration for a specific application. This study sheds light on how the accuracy of geostationary orbit determination is influenced by the duration of the observation period. The analysis presented herein aids in selecting the best-suited duration for a given application. Additionally, insights into enhancing accuracy within shorter observation durations are explored, highlighting the potential adjustments of other parameters to achieve improved precision. Observations at multiple times of day can improve accuracy: Observing the satellite at different times of day can help to average out any diurnal variations in the satellite's orbit [14, 15].

Short arc orbit determination involves using a relatively small amount of data collected over a short period of time (typically a few hours) to estimate the satellite's orbit. The goal of short arc orbit determination is to quickly determine the satellite's position and velocity so that any necessary adjustments can be made to keep the satellite in its designated orbit.

On the other hand, full cycle orbit determination involves using data collected over a much longer period (typically several days or even weeks) to determine the satellite's orbit. Full cycle orbit determination aims to produce a highly accurate model of the satellite's orbit that can be used for longer-term orbit predictions and maneuver planning.

The main difference between short arc and full cycle orbit determination is the data amount used and the accuracy level achieved. Short arc orbit determination can be completed quickly and provides a rough estimate of the satellite's position and velocity, but may not be as accurate as full cycle orbit determination. Full cycle orbit determination requires more time, data and computational resources, but can produce a highly accurate model of the satellite's orbit that can be used for precise orbit predictions and maneuver planning. In practice, satellite operators may use both short arc and full cycle orbit determination techniques depending on their specific needs and the available data. Short arc orbit determination is often used for initial orbit determination and for monitoring the satellite's position and velocity in real time, while full cycle orbit determination is typically used for longer-term orbit predictions and maneuver planning. The optimal duration will depend on the specific requirements of the application and the available resources. Performance assessment of satellite observations can be classified into optical observations, radio observations, and radio interferometry. In order to determine the orbit of a satellite, it is necessary to measure its position and velocity, which can be obtained from ground-based tracking systems or sensors onboard the satellite. There are several methods for determining a satellite's orbit, each with its own advantages and disadvantages. The choice of method and system depends on the specific needs and goals of the operator.

2. METHODOLOGY AND COLLECTING SATELLITE TRACKING DATA

Short-Arc Orbit Determination (SAOD) is a method used to estimate the orbital parameters of a geostationary satellite using a limited amount of observational data. It involves using a small set of observations (usually less than one orbit's worth) to determine the initial position and velocity of the satellite. This information is then used to propagate the orbit and refine the parameters. SAOD is typically used for satellites in the early stages of their mission when precise orbit information is not yet available.

Sat1 and Sat2 are currently operational satellites, and the data utilized in the AER and RNG methods consist entirely of authentic measurements obtained from these satellites. This study employs two distinct observation approaches to establish the orbital parameters of Sat1 and Sat2. The first method involves traditional angle and range observations, known as AER (Azimuth-Elevation-Range), while the second method is the recently developed Range-to-Range technique. AER-based orbit determination utilizes measurements of the satellite's position relative to a ground-based tracking station, involving the measurement of azimuth angle, elevation angle, and range (distance) from the tracking station to the satellite. These measurements are used to calculate the satellite's position in a reference frame that is tied to the ground station.

Range-to-Range orbit determination involves using range and/or turn-around measurements from two or more ground-based tracking stations, which are far from each other, to determine the position of the satellite.

Measurement data was gathered through electromagnetic wave propagation via a ground-based tracking system employing two distinct approaches. The first method adopted the conventional approach, capturing azimuth, elevation, and range data through large, mono-pulse tracking ground stations GS1 and GS2. The second method involved a range-to-range technique using 2.4 m non-tracking ground antennas RS1, RS2, alongside 1.8 m non-tracking remote antennas RS1’ and RS2’. For Sat1, the combination of antennas utilized was GS1 and RS1/RS1’, while for Sat2, GS2 and RS2/RS2’ antennas were used. The 48-hour measurement outcomes were employed throughout this study as a benchmark for both approaches.

2.1. Azimuth, Elevation, Range (AER) Method

The Azimuth-Elevation-Range (AER) method determines the satellite's position in terms of azimuth, elevation, and range from a ground-based tracking station. In the context of geocentric latitude for simplicity, where r and r_s represent the position vectors of the satellite and the observer's location is described in the Earth-fixed system, we can express the components of the satellite's position in the topocentric system as (x_t, y_t, z_t). This framework commonly uses two angles: azimuth (Az) and elevation (El). These angles can be calculated using the following Equation 1,

$$El = arcsin\frac{z_t}{r_t},\ and\ Az = arctan\frac{x_t}{y_t} \tag{1}$$

These angles, azimuth, and elevation provide a convenient way to describe the relative positions of the satellite and the ground station. The range component from the ground station to a satellite is incorporated along with these two angles.

The latitude, longitude, and altitude information of the ground station in the geographic coordinate system are expressed in the WGS84 standard, while the x, y, z values of the ground station in the ECEF coordinate system are calculated using Equations 2, 3, and 4.

$$x = (R_n + h)cos\phi cos\lambda \tag{2}$$

$$y = (R_n + h)cos\phi sin\lambda \tag{3}$$

$$z = ([1 - e^2]R_n + h)sin\phi \tag{4}$$

where; ϕ: geodetic latitude, λ: longitude h: ellipsoidal height, mean sea level height e: ellipsoid eccentricity, a: earth model semi major axis (m).

R_N can be expressed using Equation 5,

$$R_N = \frac{a}{\sqrt{1-e^2 sin^2\emptyset}} \tag{5}$$

In addition to this data, the instantaneous positions of satellites in space within the framework of the variables mentioned above can be obtained using classic azimuth, elevation, and ranging data, as well as observation data obtained from the ground station latitude, longitude, and altitude where measurements are made. The satellite's altitude can be obtained using Equation 6, its latitude using Equation 7, and its longitude using Equation 8.

$$h_{sat} = \sqrt{(r_e + h_g)^2 + d^2 + 2(r_e + h_g)dsin(\alpha)} \tag{6}$$

$$L_{sat} = \arcsin\,(\sin(L_g)\,cos(\theta) + \cos(L_g)\,sin(\theta)cos(\beta) \tag{7}$$

$$\lambda_{sat} = \arcsin\,(-\frac{sin(\theta)sin(\beta)}{\cos(L_g)}) + \lambda_g \tag{8}$$

where: $h_{sat,}$ altitude of satellite (m), L_{sat} : satellite latitude (radian), L_g: ground station latitude and λ_{sat}: satellite longitude (radian), d: distance between the ground station and the satellite (m), α : azimuth angle (radian), β: elevation angle (radian), λ:longitude Θ: geodetic angle.

2.2. Range Method (RNG) Method

In the field of orbit determination, a fundamental measurement involves determining the distance between a ground station, an earth-based instrument used for tracking, and a satellite in orbit. We use the position vectors of the ground station r_g and the satellite r_s to quantify this distance. The ideal range, denoted as ρ, represents the scalar magnitude of the position vector pointing from the ground station to the satellite. Mathematically, it is expressed in Equation 9,

$$\rho = [(r_s - r_g).(r_s - r_g)]^{1/2} \tag{9}$$

Moreover, when we have an observed range ρ_{obs} at a specific time (t), and the true position vectors r_s and r_g at that time, the observed range ρ_{obs} is related to the ideal range ρ_o with the addition of ϵ , which accounts for instrumental errors and propagation delays. This relationship is in Equation 10,

$$\rho_{obs} = \rho_o + \epsilon \tag{10}$$

It's important to note that the geometric range ro remains consistent regardless of the choice of axes used to describe the positions of r_s and r_g. In simpler terms, if we represent the system with Earth-fixed coordinates (x, y, z), the geometric range can be calculated in Equation 11,

$$\rho = [(x - x_g)^2 + (y - y_g)^2 + (z - z_g)^2]^{1/2} \tag{11}$$

This method is essential for precisely determining the distance between a ground station and a satellite, providing crucial information for orbit determination.

2.3. GEO Satellite Dynamic Model

This study employs a dynamic model for communication satellites, representing the satellite's position and velocity dynamics in the J2000 Earth-Centered Inertial (ECI) coordinate system. In the J2000 system, the x and z axes align with the mean vernal equinox and the mean rotation axes of the Earth at a specific reference time: 1 January 2000, 12:00:00.00 UTC (J2000 = 2000, January 1.5 = JD - Julian date - 2451545.0).

The position and velocity vectors of the satellite are described by the following Equation 12,

$$r = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, v = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \tag{12}$$

With advancements in computer technology, the utilization of numerical methods to model satellite motion in orbit is rapidly expanding. Numerical methods offer the flexibility to integrate any perturbing force at any stage of the simulation, accomplished by augmenting these forces to Cowell's two-body equation formulation. Consequently, the total acceleration of the satellite can be computed using Equation 13,

$$a = -\frac{\mu}{r_e} r + a_{perturbed} \tag{13}$$

where: μ :Earth gravitational constant, r_e :Earth Radius r: satellite position vector.

$a_{perturbed}$ represents the additional forces acting on the satellite, including non-gravitational effects such solar radiation pressure, gravitational influences from other celestial bodies, and tidal forces. These perturbing forces introduce deviations from the simplified two-body dynamics described by the gravitational force alone, resulting in a more precise modeling of satellite motion in orbit.

Once the mathematical model representing a satellite, whether in a geostationary or low Earth orbit, is established, the equations derived can be solved using various methodologies. These methodologies encompass a spectrum of techniques, including the Kalman filter, least squares error analysis, and Monte Carlo approaches, among others. Each method offers distinct advantages and applications in orbit determination [16, 17].

2.4. Orbit Determination by Iterative Least-Squares Estimation

The Iterative Least-Square Estimation determines the orbits based on different observation intervals. This method is mainly based on an iterative process: on each step, the best-fitted curve or line trying to be found by reducing the squares of the residuals on the used point of the curve. In short, the orbital parameters are trying to be found by iteratively minimizing the residuals of the measurement data.

The goal is to determine the initial conditions that minimize a certain mathematical function called J. The function J is a sum of squares of differences between observed values and computed values as shown in Equation 1, and it depends on a vector X that represents the initial position and velocity of the satellite [18, 19] as shown in Equation 14.

$$J = \sum_{j=1}^{l} (O_j - C_j)^2 \tag{14}$$

where O_j represents the measurement data of satellites orbit, C_j represents the corresponding computed orbital data, which is determined using a reference value X_j and an appropriate observation-state model for the measurement. The symbol j represents the total number of observations.

To be more precise, the computed observation C_j is based on a reference set of coordinates for both the tracking system and the satellite. The difference between the observed value O_j and the computed value C_j is known as the observation residual and is denoted as O_j, C_j. To find the values of X that minimize j, the partial derivatives with respect to $X(t_0)$ must be zero. This leads to a set of nonlinear algebraic equations as shown in Equation 8, 9 and 10.

$$X(t_0)_{n+1} = X(t_0)_n + (H_n^T H_n)^{-1} H_n^T y_n \tag{15}$$

$$\acute{x}_{n+1} = X(t_0)_{n+1} - X(t_0)_n \tag{16}$$

$$\acute{x}_{n+1} = (H_n^T H_n)^{-1} H_n^T y_n \tag{17}$$

The least squares approach utilized in this study, widely recognized among satellite operators, facilitates a gradual accumulation of measurements, enabling a sequential formulation of the method. The iterative process for a given set of observations can be repeated until a convergence criterion is met, or it falls below a predetermined threshold [18]. To assess the impact of the sole time interval (measurement duration), the least squares approach was maintained consistent across all calculations.

The algorithm described below is employed for orbit calculations using the AZEL method [20].

AER method

```
function AER to Position(Az, El, R)
//Convert Azimuth and Elevation to ECEF or ECI coordinates, use ground station known Latitude, Longitude and Altitude and Greenwich Sideral Time, GST
X, Y, Z = AzEl-to-ECEF(Az, El, R)
//Use iterative least squares estimation for refinement for iteration in range (max iterations)
//Calculate predicted Azimuth, Elevation, and Range based on current X, Y, Z
predicted_{Az}, predicted_{El}, predicted_{R} = Position-to- AzEl(X, Y, Z)
//Calculate residuals (difference between observed and predicted values)
 residual_{Az} = Az - predicted_{Az}, residual_{El} = El - predicted_{El}, residual_{R} = R - predicted_{R}
 // Update X, Y, Z using LSE adjustments
X += LSE_adjustment(residual_{Az}, residual_{El}, residual_{R})
Y += LSE_adjustment(residual_{Az}, residual_El, residual_{R})
Z += LSE_adjustment(residual_{Az}, residual_{El}, residual_{R}) \\
// Check for convergence
if convergence criteria met (residual_{Az}, residual_{El}, residual_R) then break
 return X, Y, Z
```

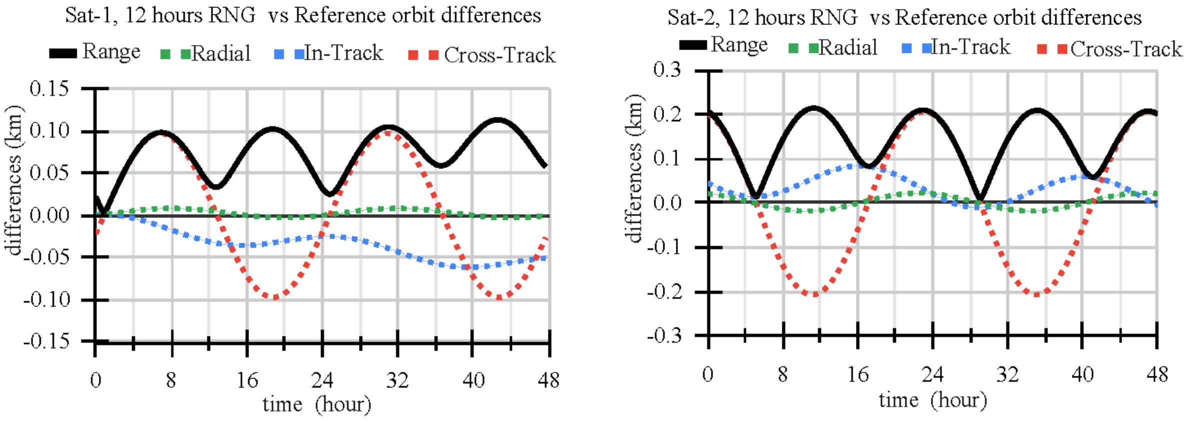
The orbit determination process using the RNG method follows a similar algorithm to that of the AER method, but to avoid redundancy, the specific details are not provided here. In both methodologies, two distinct scenarios are considered: one where existing bias values are used, and the other where bias values are recalculated to obtain new values.

3. RESULTS

This study aimed to evaluate the accuracy of short arc orbit determination for GEO satellites using two data collecting methods, namely azimuth elevation range (AER) and range-to-range (RNG), and to compare their accuracy in determining satellite orbits.

The orbit accuracy of Sat1 and Sat2 located at different orbits was assessed using the AER and Range-to-Range data collection methods. Iterated Least Squares was utilized to determine the satellite orbit for 6-hour to 48-hour period, incorporating new bias calculation and pre-existing bias values. The orbital parameters coming from the 48-hour period of the measurements were used as a reference.

There are many charts available to visualize the impact of orbit determination methods (AER and RNG) and station bias values on satellite orbit accuracy. Among them, Figure 1 is selected as well-suited samples. Figure 1.a) displays the in-track, cross-track, radial and 3D Euclidean distance between the orbits obtained using short arc 12-hour measurement data for Sat1 and the 48-hour measurement value for Sat1. Similarly, Figure 1.b) shows the distances between the orbits obtained using short arc 12-hour measurement data for Sat2 and the 48-hour measurement value for Sat2. Both orbit determinations use RNG method and pre-existing bias data.



a) b)

Figure 1. RMSE difference between a) Sat1 orbit obtained using 12-hour short arc data and the reference, b) Sat1 orbit obtained using 12-hour short arc data and the reference.

between Sat2 orbit obtained using 12-hour short arc data and reference. (AER with pre-existing bias data)

As demonstrated in Figure 1.a), the maximum root mean square error (RMSE) between the reference 48-hour measurement and 12-hour measurement is about 0.1 km for Sat1 and 0.2 km for Sat2, resulting in valid orbit results for both satellites.

Table 1 displays the root mean square error (RMSE) estimation for orbit determination across four distinct evolutionary types, employing existing bias values, i.e., without new bias calculation (GS1 AER Sat1, RS1-RS1’ RNG Sat1, RS2-RS2’ RNG Sat2, GS2 AER Sat2) for different time intervals (6, 12, 18, 24, 30, 36, and 42-hour).

For all the satellites, the RMSE decreases with an increase in the time interval, except for GS2 AER Sat2 where the RMSE increases with an increase in the time interval.

Table 1. In-flight RMSE values of estimated orbit for 2 satellites and 2 different orbit determination methods (using pre-existing bias).

Sat/ Stations/ Obs Types	06h	12h	18h	24h	30h	36h	42h
Sat1 GS1 (AER)	1.384	1.372	0.853	0.427	0.267	0.322	0.212
Sat1 RS1-RS1’ (RNG)	0.649	0.079	0.036	0.199	0.056	0.022	0.004
Sat2 GS2 (AER)	0.436	0.974	0.927	0.863	1.025	0.985	1.048
Sat2 RS2-RS2’ (RNG)	0.764	0.154	0.100	0.056	0.026	0.057	0.014
Average	0.8082	0.6447	0.4791	0.3860	0.3435	0.3467	0.3261
Avg AER	0.9099	11.731	0.8902	0.6448	0.6458	0.6537	0.6302
Avg RNG	0.7065	0.1164	0.0681	0.1272	0.0412	0.0397	0.0220

For the shortest time interval (6-hour), GS1 AER Sat1 has the highest RMSE value (1.384 km) followed by RS2-RS2’ RNG Sat2 (0.764 km), GS2 AER Sat2 (0.436 km), and RS1-RS1’ RNG Sat1 (0.649 km).

For the longest time interval (42-hour), RS1-RS1' RNG Sat1 has the lowest RMSE value (0.004 km) followed by GS1 AER Sat1 (0.212 km), RS2-RS2’ RNG Sat2 (0.057 km), and GS2 AER Sat2 (1.048 km).

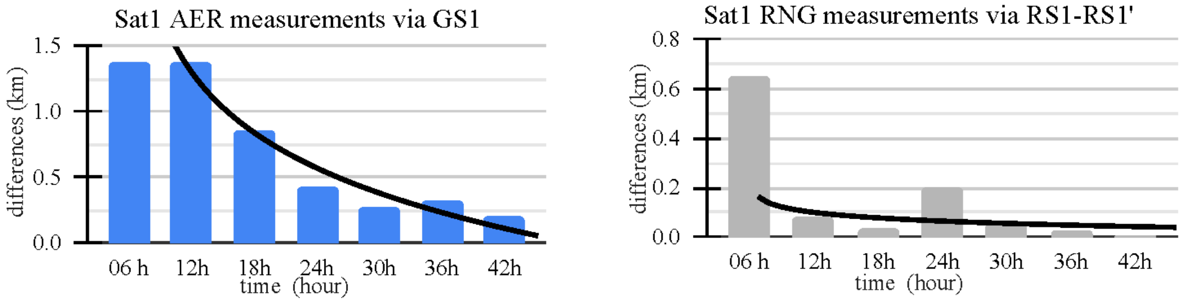
Table 2 shows the Root Mean Square Error (RMSE) values for two satellites and two orbit determination methods (AER and RNG) using new bias calculation. The table displays the differences in km from the reference 48-hour measurement for various time intervals ranging from 6 to 42- hour.

For Sat1, the RNG method shows lower RMSE values than the AER method for all time intervals, with the lowest RMSE value of 0.036 km achieved for 18-hour measurement using the RNG method. For Sat2, the RNG method also shows better results than the AER method, with the lowest RMSE value of 0.028 km achieved for a 42-hour measurement using the RNG method.

Table 2. In-flight RMSE values of estimated orbit for 2 satellites and 2 different orbit determination methods (with new bias calculation).

Sat/ Stations/ Obs Types	06h	12h	18h	24h	30h	36h	42h
Sat1 GS1 (AER)	54.760	234.921	75.381	35.578	4.031	0.318	0.193
Sat1 RS1-RS1’ (RNG)	28.541	18.485	0.036	4.178	0.051	0.333	0.106
Sat2 GS2 (AER)	35.044	0.211	0.105	0.207	0.051	0.173	0.028
Sat2 RS2-RS2’ (RNG)	96.617	2.409	4.177	0.753	0.781	0.587	0.215
Average	53.741	64.006	19.925	10.179	1.229	0.353	0.136
Avg AER	75.688	118.665	39.779	18.166	2.406	0.453	0.204
Avg RNG	31.793	9.348	0.071	2.192	0.051	0.253	0.067

The results show that the RNG method generally provides lower RMSE values compared to the AER method for both satellites, indicating better accuracy in orbit determination. Additionally, the choice of ground station and observation method can affect orbit determination accuracy. The average RMSE values decrease as the time interval increases, indicating that longer measurement times can improve the accuracy of short arc orbit determination. However, upon examining Table 1, anomalies are observed in the 36-hour column for both 'Sat1 GS1 (AER)' and 'Sat2 GS2 (AER)'. This trend is consistent with the values presented in Table 2. These minor deviations are attributed to potential measurement errors. It is important to note that these glitches do not significantly impact the overall performance of the method.

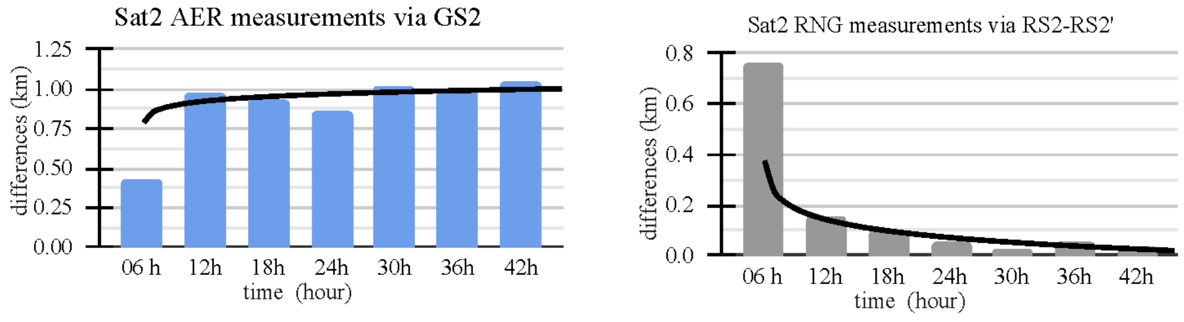


a)

b)

Figure 2. Sat1 RMSE values for different measurement length a) AER method from GS1 and b) RNG-RNG method from RS1-RS1'(determined no new bias calculation during process).

Figure 2.a) illustrates the calculated orbits trend line for Sat1 using AER method without new bias calculation differences from the 48-hour reference results, while Figure 2.b) shows the trend line using RNG method. The results suggest that the RNG method generally provides higher accuracy in orbit determination, as indicated by lower RMSE values, compared to the AER method. Moreover, the accuracy of orbit determination can be affected by the selection of ground stations and observation method.

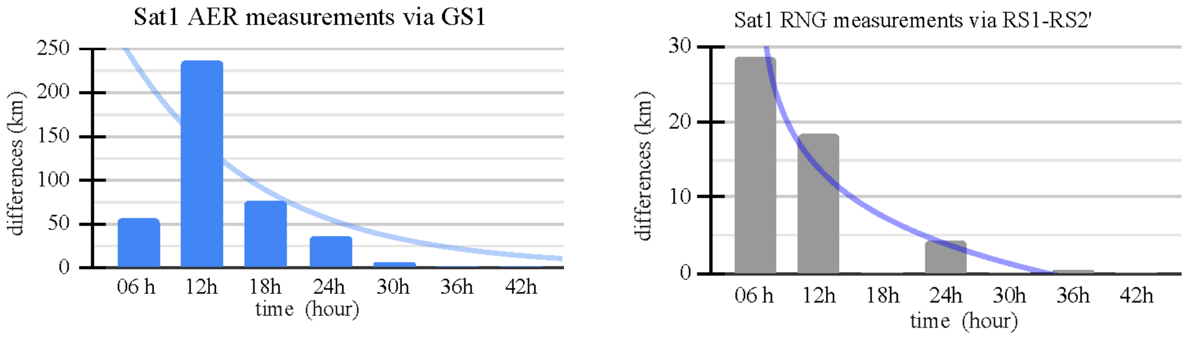


a)

b)

Figure 3. Sat2 RMSE values for different measurement length a) RNG-RNG method from RS2-RS2' b) AER method from GS2 (using pre-existing bias during the process).

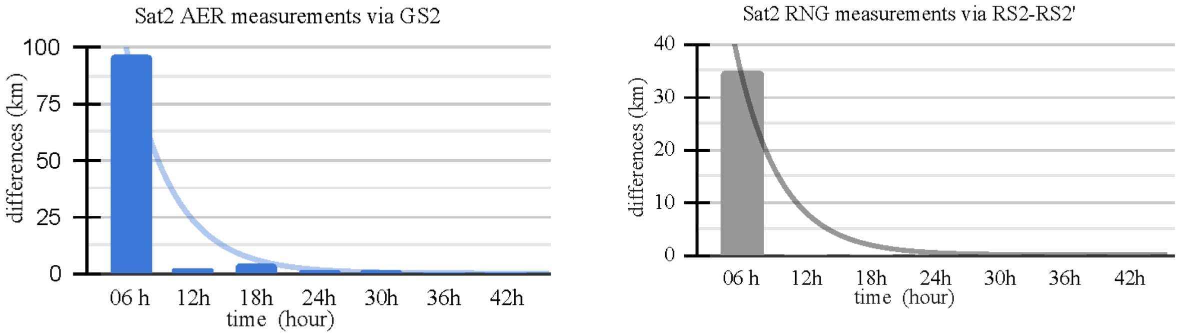
Figure 3.a) shows the trend line of calculated orbits for Sat2 using the AER method without new bias calculation differences from the 48-hour reference results. Figure 3.b), however, shows the trend line using the RNG method. It is worth noting that Figure 3a shows slightly worse accuracy with increasing time, contrary to expectations. However, this decrease in accuracy is small and is assumed to be due to errors in the input observation data.



a) b)

Figure 4. Sat1 RMSE values for different measurement length a) AER method from GS1 and b) RNG-RNG method from RS1-RS2’ (new bias calculated during process).

Figure 4.a) depicts the calculated orbit trend line for Sat1 using 6 to 42-hour observation data and AER method with new bias calculation differences from the 48-hour reference results. On the other hand, Figure 4.b) illustrates the calculated orbit trend line for Sat1 using 6 to 42-hour observation data and RNG method without new bias calculation differences from the 48-hour reference results. The findings suggest that the RNG method generally produces lower RMSE values than the AER method, indicating better accuracy in orbit determination. It is important to note that the selection of the ground station and observation method can impact orbit determination accuracy.

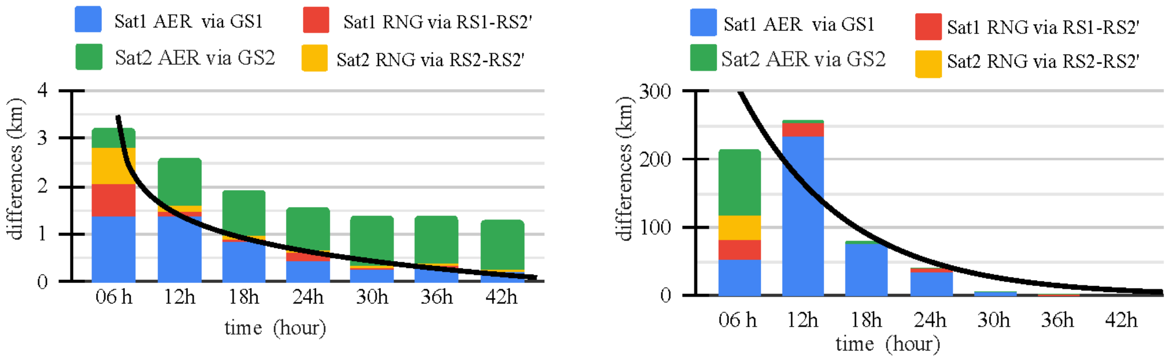


a) b)

Figure 5. Sat2 RMSE values for different measurement length a) AER method from GS2 and b) RNG-RNG method from RS2-Rs2’ (new bias calculated during process).

Figure 5.a) and Figure 5.b) illustrate the trend lines of determined orbit accuracy for Sat2 using both AER and RNG methods with new bias calculation during the process. The results

indicate that the RMSE values from the RNG method are generally better than those of the AER method when compared to the reference 48-hour result.



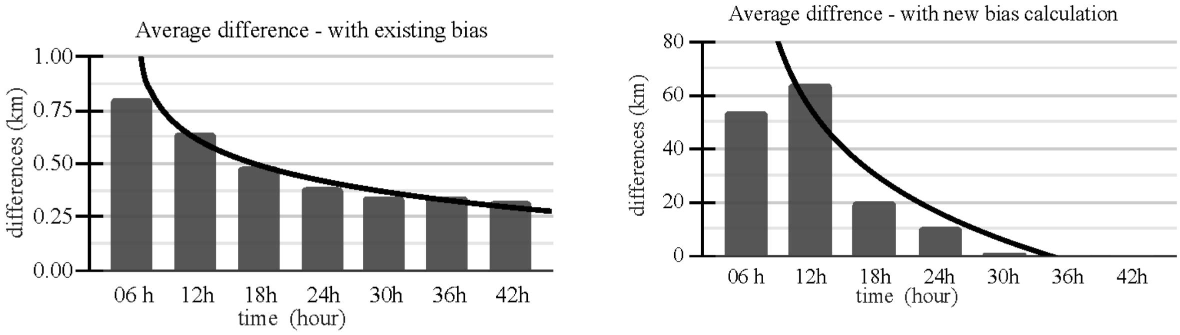
a)

b)

Figure 6. Sat1 and Sat2 cumulated RMSE values for different measurement length together with AER and RNG-RNG method (using pre-existing bias during the process) b) the same calculation including new bias in the process.

Figure 6.a) displays the cumulated RMSE values for Sat1 and Sat2 using both AER and RNG methods. The colored bars represent the error contribution of each method to each satellite without new bias calculation. On the other hand, Figure 6.b) shows the same approach with the inclusion of the new bias calculation process.

Figure 7 illustrates the overall performance of both the AER and RNG methods through trendlines. Figure 7.a) shows the average trend line of calculated orbits for Sat1 and Sat2 using the AER and RNG methods without new bias calculation. The x-axis represents the time period, while the y-axis represents the orbit accuracy in terms of Root Mean Square Error (RMSE) values. The average trend lines for the AER and RNG methods are shown in black color. Figure 7.b) shows the same results but includes bias calculation. The black trend line indicates the results with new bias calculation, and the bars indicate the average differences in hour basis. Both figures indicate that accounting for biases during the orbit determination process leads to decreased accuracy in the determined orbit.



a) b)

Figure 7. Sat1 and Sat2 Average RMSE values for different measurement length together with AER and RNG-RNG method (using pre-existing bias during the process) b) the same calculation including new bias in the process.

In our study, we examined the effect of the observation duration on the accuracy of geostationary orbit determination. We collected observations of two geostationary satellites over a range of periods, from 6-hour- to 42-hour, and used these observations to calculate the satellite's orbit using standard orbit determination techniques.

According to the findings, when pre-existing bias values are used, the average root mean square error (RMSE) values for AER and RNG observation data types are 0.792 km and 0.160 km, respectively. However, when new bias calculation is utilized, the average RMSE values for AER and RNG observation data types increase significantly to 36.480 km and 6.254 km, respectively.

The Range-to-Range (RNG) method, employing pre-existing bias values, demonstrates sufficient accuracy for satellite operators to maintain control over the satellite within the designated control window during 6-hour and 12-hour observation intervals. On the other hand, the Azimuth-Elevation-Range (AER) method, utilizing existing bias calculations, necessitates an extended 18-hour observation duration for achieving nominal operational accuracy. Notably, recalculating bias values for the RNG method during an 18-hour observation period reduces the required observation duration for AER to 30-hour to achieve the desired nominal operational accuracy. This highlights the adaptability and efficiency of the RNG method in optimizing the trade-off between observation duration and operational accuracy.

However, using extremely long observation periods may not always be practical or necessary, and a trade-off between accuracy and other factors such as cost and operational constraints may be necessary.

The findings of this study align with existing literature in the field, including research on satellite orbit determination utilizing batch filtering with unscented transformation, short-arc orbit determination methods for GEO satellites, and differential evolution algorithms for initial orbit determination. These prior studies, as referenced [21, 22, 23, 24], reinforce the appropriateness and reliability of the methods and approaches employed in this research. The consistency observed with previous findings underscores the validity of the study's conclusions and underscores its significance. Moreover, the concordance between our results and existing literature enhances the credibility of our research and encourages further exploration of the topic.

This result highlights the intricate trade-offs between observation duration and accuracy in geostationary orbit determination. While a longer observation period may offer improved accuracy, it is crucial to recognize that this relationship is influenced by various factors, including whether the observation arc crosses the orbit node. Thus, selecting the optimal observation period for a specific application necessitates a comprehensive consideration of these factors. Our study underscores the significance of balancing observation duration with these factors in geostationary orbit determination, emphasizing that longer periods may not always be feasible or result in proportionate gains in accuracy.

4. CONCLUSION

Short-arc orbit determination is essential for determining the orbits of GEO satellites, particularly when there is a need to balance precision with issues like cost and operational limitations. This strategy is especially important in the initial stages of satellite launch, when time restrictions restrict the availability of orbit data. Short arc approaches are more practical than full cycle orbit determination. They are suitable for situations when little tracking data is enough for initial placement or necessary corrections, without the need for continuous and protracted tracking. This paper provides significant contributions to the current body of literature by presenting insights into the efficacy of short arc orbit determination methods, namely through a comparative analysis of the Range-to-Range (RNG) and Azimuth-Elevation-Range (AER) approaches. Our study's results highlight the RNG technique's exceptional precision, establishing it as a more advantageous option for calculating orbits.

Furthermore, the study highlights the effectiveness of using pre-existing bias values in the RNG approach, demonstrating its practical usefulness and cost-efficiency in specific situations. This study enriches our comprehension of short arc orbit determination, offering significant contributions to both theoretical understanding and practical implementations in the field. Potential future advancements of this research could investigate its applicability to complete electric satellite maneuver operations, which involve frequent adjustments to the orbit and shorter time intervals between consecutive firings, posing distinctive obstacles and possibilities.

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To Cite This Article: U.C. Yılmaz, Ü. Güler, C. Şakacı, İ. Öz, *Influence of Time Intervals: Comparative Analysis of Short-Arc Orbit Determination for GEO Satellites Using AER and RNG Methods*, Journal of Aeronautics and Space Technologies **17**(2), 1-17 (2024).

VITAE

Cezmi Yılmaz obtained his Bachelor of Science degree in Electrical-Electronics Engineering and subsequently contributed as a research assistant during his pursuit of a master's degree. Venturing into Türk Telekom's Satellite Control Center Directorate, he commenced his professional journey as an engineer and gradually ascended to the esteemed position of Satellite Control Engineer. Driven by a fervent interest in orbital dynamics and Low Earth Orbit (LEO) as well as Geostationary Orbit (GEO) communications, he pursued doctoral studies, delving deeper into these domains. Within Turksat, he undertook pivotal roles, notably as the Director of the Satellite Control Center. His active involvement in the procurement of Turksat satellites underscores his dedication to advancing satellite technologies and their operational efficiency.

Ümit Güler holds a Bachelor of Science degree in Electronics Engineering. His career trajectory began as a satellite control engineer at Turk Telekom before seamlessly transitioning to a similar role at Turksat. Currently, he presides over the crucial position of Satellite Control Stations Director at Turksat. Noteworthy is his integral participation in the Turksat procurement project, contributing significantly to the enhancement of satellite operations.

Cemal Şakacı academic pursuit led to the acquisition of both Bachelor's and Master's degrees in Aeronautical Engineering. He commenced his professional journey contributing his expertise as a satellite control engineer at Turk Telekom and later at Turksat AŞ. Presently, his role at Turksat AŞ encompasses the crucial responsibilities of a Flight Dynamics Engineer, where his expertise continues to play a vital role in optimizing satellite operations.

İbrahim Öz obtained his B.Sc. degree in Electrical-Electronics Engineering and served as a research assistant during his master's studies. Subsequently, he joined Türk Telekom's Satellite Communications Center Directorate as an engineer, eventually assuming the role of Communication Monitoring Manager. His doctoral studies focused on electronic science, specifically video compression techniques for satellite broadcasting. Within Türksat AŞ, he held several positions, including Director of Satellite Control and Monitoring and Deputy General Manager. He actively participated in satellite procurement, control and monitoring, design, production monitoring, insurance, and technology transfer. He made significant contributions to the development and launch of various Türksat satellite programs. In addition, he held key positions contributing to the production of a domestic communication satellite, ground stations and infrastructure at Türksat AŞ. Furthermore, he serves as a part-time lecturer, delivering courses on Satellite Technologies, Satellite Operations, Satellite Communications, Introduction to Electrical and Electronic Engineering, and Spacecraft Engineering and Design at Eskişehir Technical University, Gazi University, and Ankara Yıldırım Beyazıt University.