



## RESEARCH ARTICLE

# Modulation Transfer Function Extraction and Parametric Effects

## Modülasyon Aktarım Fonksiyonunun Çıkarımı ve Parametrik Etkiler

Kamil B. Alıcı<sup>1\*</sup>

<sup>1</sup> TÜBİTAK Space Technologies Research Institute, 06800, Ankara, Türkiye

**Received:** November 7, 2023

**Revised:** January 15, 2024

**Accepted:** February 13, 2024

### Abstract

We compute the optical transfer function and corresponding satellite image estimate in order to test validity of our modulation transfer function (MTF) extraction algorithms. The test targets allow MTF computation by using slanted edge and sine wave star patterns. Comparison of extracted MTFs with the input ones yields 0.013% mean squared error for the slanted edge method and less error for the sine wave star method. Various factors affect the extracted MTF results such as slanted edge angle, noise level, image restoration level, region of interest selection and oversampling factor. We investigated effect of each factor on the MTF extraction performance.

**Keywords:** Optical Transfer Function, Modulation Transfer Function, Image Chain Simulation, Imaging, Image Sensors, Image Sampling, Image Resolution, Image Restoration, Wiener Filtering

### Öz

Modülasyon aktarım fonksiyonu çıkarım algoritmalarımızın geçerliliğini test etmek için optik aktarım fonksiyonlarını ve ilgili görüntü öngörüsünü hesaplıyoruz. Test görüntüsü hedefleri, MTF hesabı için eğik kenar ve sinus dalgası desenlerini içermektedir. Girdi olarak kullanılan MTF değerleri ile çıkarım sonucu elde edilen MTF değerleri karşılaştırıldığında, eğik kenar yöntemi için %0,013 ortalama kare hatası elde edilmiştir, sinus dalgası yöntemi için bu hata daha da azdır. Çeşitli faktörler MTF çıkarım değerlerini etkilemektedir. Bunlar, eğik kenar açısı, gürültü seviyesi, görüntü onarımı seviyesi, ilgilenilen alan seçimi, ve aşırı örnekleme faktörüdür. Her bir faktörün MTF çıkarım başarımına etkisini inceledik.

**Anahtar Kelimeler:** Optik Aktarım Fonksiyonu, Modülasyon Aktarım Fonksiyonu, Görüntü Zinciri Simülasyonu, Görüntüleme, Görüntü Sensörleri, Görüntü Çözünürlüğü, Görüntü Onarımı, Wiener Filtresi

## 1. INTRODUCTION

One of the image quality studies at the satellite deployment phase is the extraction of modulation transfer function from specifically selected satellite based images. However, accuracy, precision and robustness of the extraction methods are limited due to the factors such as image noise, signal level, sampling, and utilized area of interest in analyses. Investigation of such factors and quantifying their effects helps to develop better MTF extraction processes and interpretation of MTF results.

Image quality estimation methods have been developed by using the imaging chain approach [1, 2]. Image restoration processes enable us to improve the quality of degraded satellite images. Overall optical transfer function (OTF), which forms the key input for

\*Corresponding Author  
E-mail: [kamil.alici@tubitak.gov.tr](mailto:kamil.alici@tubitak.gov.tr)

image restoration process, describes how imaging device and imaging environment affect the final images. Image restoration algorithms strongly depend on the OTF representation of the system [3]. Cameras that are not digital require different form of OTF estimation methods [4, 5]. Digital cameras, on the other hand, require extra methods for accuracy improvement. One of such methods is the standard slanted edge method that provides spatial frequency response (SFR) [6]. SFR is a general term and it is equal to MTF for sine wave input and contrast transfer function for square wave input [7]. Accurate determination of SFR is critical for many applications [8].

Placement of focal plane at the optical focus point is very important in imaging performance [9]. Extracted MTF values could provide a measure for focusing quality. In the slanted edge method, super sampling is required for better accuracy and precision [10]. MTF extraction was typically performed by using various target patterns [3-5, 11, 12]. Slanted edge method provided accurate SFR values for image acquisition devices. For film camera type devices continuous images were oversampled. On the other hand, for digital devices sampling introduces errors in SFR estimation [3]. For spatially sampled imaging devices, one needs to alter the classical slanted edge technique. Spatial resolution of a sampled imaging system is its ability to distinguish finely spaced points. MTF gives an idea on the spatial resolution of an imaging system. Standard slanted edge method has some limitations [6].

In order to measure performance of an optical system, point spread function (PSF) may also be used instead of MTF. PSF provides the best focal position and describes the spatial spread of a point source at the focal plane [2]. MTF is the response of optical systems to sinusoidal modulation and this response can be derived from PSF via Fourier transform. One of the patterns utilized for MTF extraction is the modulated Siemens star. Multidirectional MTF of systems can also be measured by another method [13]. Edge of a rectangular region of interest was projected to horizontal axis that yields highly sampled data for the slanted edge case. Number of points per pixel area gives the oversampling factor. Oversampling reduces extraction artifacts of MTF curves. Nonuniformity of oversampling points is a source of error. Slanted edge method provides information on the spatial resolution of sampled imaging devices [10].

In the experiment setups, utilization of knife-edge targets is rather simple than fabricating sinusoidal targets. Knife-edge target image is used to extract OTF. It's mathematical representation is a step function as the edge acts like a sharp discontinuity [3]. While creating the one dimensional edge profile, rows of a two dimensional region of interest of target image is utilized [8]. This edge data is obtained from a tilted image with standard angle of  $5^\circ$  [8]. MTF curves can be extracted with these algorithms even in the presence of noise [6]. One implementation of MTF extraction is the determination of imaging systems' best focus. It is possible to compare the slanted edge test with for example PSF test, which involves mapping of a point source at the focal plane [2].

One can fit an analytical function to the one dimensional oversampled edge profile in the slanted edge method. By using this analytical function or using the direct edge profile one can obtain the line spread function (LSF) via derivative operation. MTF is the result of Fourier transform operation applied to the LSF. In the standard method, a super sampling

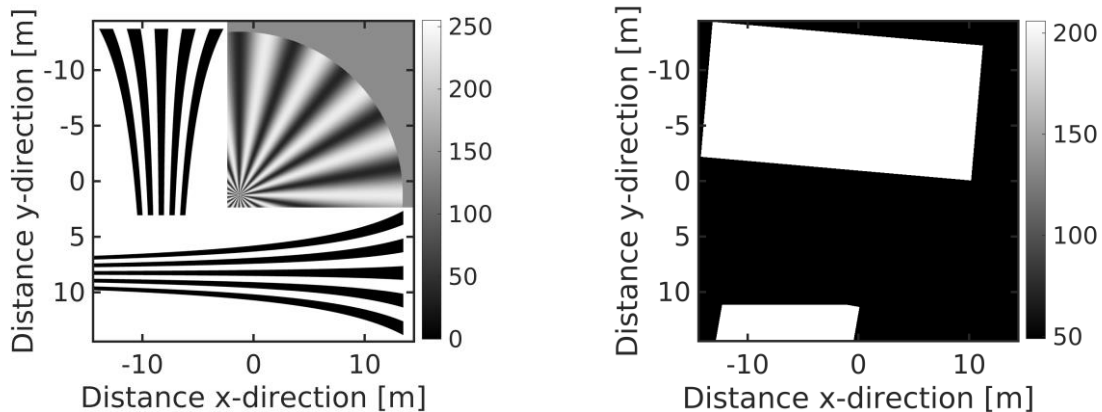
ratio of four was utilized. One can generate a regularly spaced super sampled ESF by interpolating irregularly spaced samples. Strong effects of noise can be reduced by the application of low pass filter to the reconstructed ESF. For the case of sine wave star pattern, modulation profiles at various radius values result in direct computation of MTF [13]. For LSI systems, cascaded MTF, which is the product of component MTFs, represents the system.

The traditional knife-edge method yields inaccurate results for digital imaging devices, and oversampling is required. The standard has chosen the oversampling factor as fixed with value four, which is not very well reasoned, and its stability is under question. Additionally, standard algorithms have reduced performance for real time measurements.

In order to improve the standard method, several solutions were provided [12, 14-18]. Accurate MTF estimates were incurred for undersampled cases [3]. Synthetic images provide quantitative perspective for the implementation of input MTFs. Various parametric scenarios were implemented such as edge angle, region of interest, peak SNR, modulation gain, signal bandwidth, and binning ratio. Multidirectional edge profiles of square wave star pattern were used to get oversampled curves [13]. An alternative method to the original one was proposed, which requires similar computational power [10]. These works were followed by on-orbit PSF measurements in real time [17].

Error in MTF determination could be based on fixed sampling of imaging devices [4, 5]. When we shift the points, we get different results for the case of digital image sensors. The slanted edge method allows us to increase the sampling by superimposing several scans. Finely sampled ESF can be obtained by analytic function fitting [11]. Various fitting curves were investigated in literature [12]. MTF values could be degraded by some physical mechanisms such as pixel cross talk. Slanted edge method results, and sine wave star pattern results could be compared [19]. We have extended results of our recent conference paper here [20].

Different input image patterns are useful in determination of different image quality metrics such as MTF, SNR, and resolution. Our synthetic ground truth (SGT) images cover such test patterns. Two sections of our SGT were demonstrated in Figure 1 (a) and (b). We can extract MTF from these patterns by using our slanted edge method and sine wave star method. Spatial resolution can also be extracted from these images.



(a) (b)

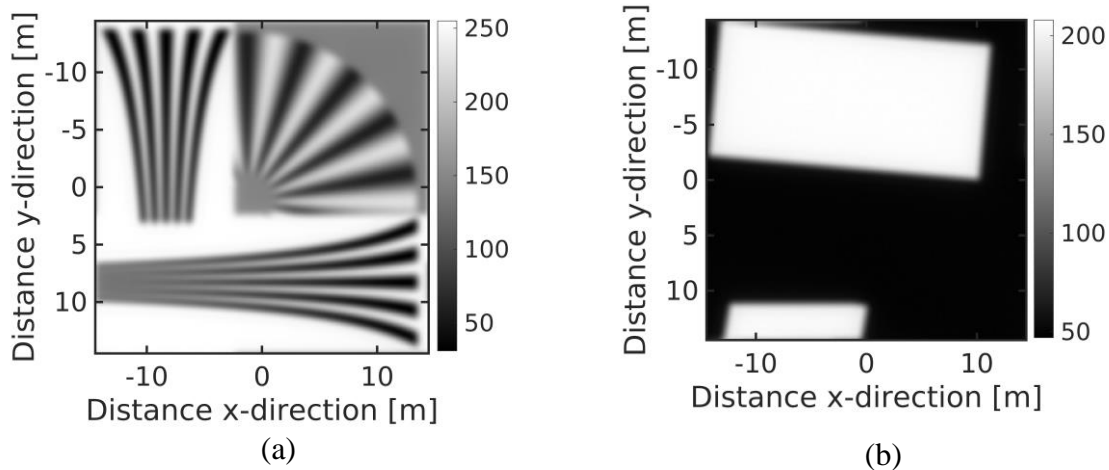
**Figure 1.** (a) Our synthetically generated ground truth image (SGT) sections. The top right pattern is partial sine wave star pattern, the top left pattern is hyperbolic wedge for across track direction, bottom pattern is the hyperbolic wedge for along track direction (b) SGT section for along track MTF extraction with slanted edge pattern.

In the present paper, several important factors that affect the MTF determination were investigated. These include dark noise, slanted edge angle, quality of image restoration, region of interest and oversampling factor.

Our algorithms applied in this study provide a robust, accurate and precise way to extract image sensor MTF. We compared input MTFs with the extracted ones and obtain very good agreement. For high SNR case the mean squared error between input and output MTFs were less than 0.013%.

## 2. METHODOLOGY

We use imaging chain approach together with our SGT sections in order to estimate the satellite images [20-22]. The method was based on convolution operation and provides the image estimate at the focal plane. Overall estimation of the OTF is more common than PSF studies. Multiplication of SGT spectrum with the OTF provides image spectrum. Fourier transform of this spectrum yields irradiance image at the focal plane. OTF of system components was computed under atmosphere, optics, image sensor and platform motion domains. To name a short list of parameters utilized in these computations we write, operation wavelengths, satellite altitude, aperture diameter, pixel geometry, pixel size, and effective focal length. In the presence of noise the case is not the ideal one, and Poisson and Gaussian distributions are utilized. Digital number values of images are obtained by considering the bit number, and full well capacity values. In Figure 2 (a) and (b) ICS images for slanted edge and sine wave star patterns are demonstrated. In these ICS studies, we do not include effects of atmosphere, optical aberrations, and several image sensor related effects.



**Figure 2.** (a) Our SGT with sine wave star pattern, blurred in accordance with several ICS effects. (b) Our SGT with slanted edge patterns degraded in accordance with several ICS effects.

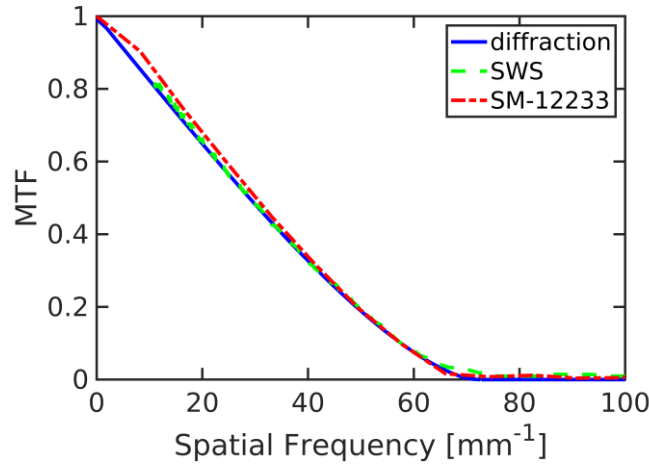
### 3. IMPLEMENTATION

MTF computation starts with extracting oversampled edge spread curve from slanted edge target's image data. Results depend on the oversampling factor and we investigated a number of oversampling factors. Image noise was transferred to the ESF and it can be reduced by using a filter such as median filter or Savitsky-Golay filter. Effect of noise reduction was investigated with different median filter windows. A possible next step is to apply fitting to the ESF in order to fully remove noise and get an analytic function for ESF.

However, this step may result in loss of information. In order to minimize such curve fitting related losses, we can either omit this step or utilize a series expansion in order to analytically model the edge function to the finest detail. The next step is the computation of line spread function (LSF) by taking the derivative of the ESF in spatial domain. We finally obtain the MTF by getting the Fourier transform of the LSF and defining the spatial frequency axis properly.

Oversampling of the slanted edge image is the first step for MTF extraction. We use the standard edge angle of  $5^\circ$  and analyze edge gradient relative to the sampling points. For the across track direction, MTF slanted edge is generated with positive clockwise direction from vertical axis. In the standard, number of points used in binning was four [13]. Region of interest that provides ESF data could be rectangular or square.

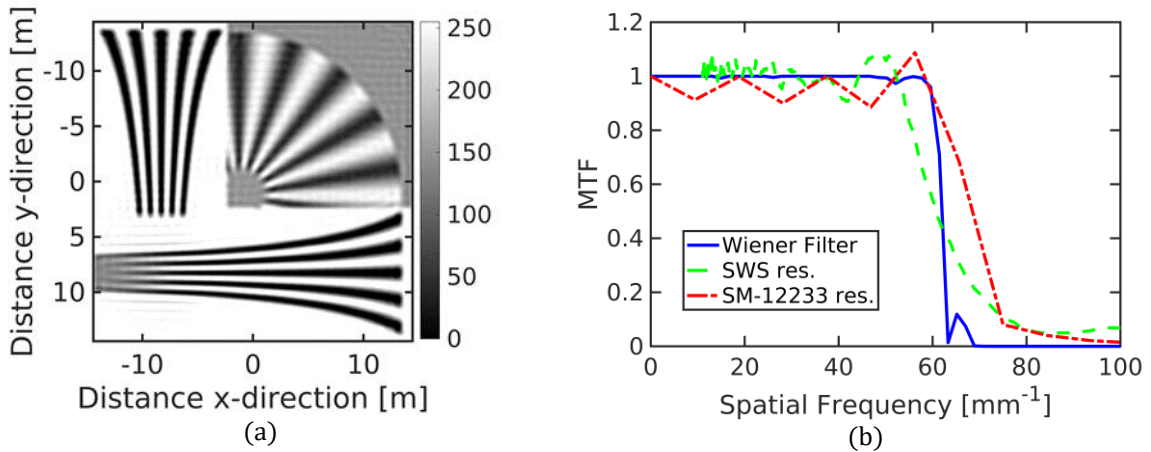
Oversampling of a slanted edge allows MTF estimation. A region of interest with square or rectangular geometry could be selected. Subpixel sized projected points in the horizontal axis enable high ESF resolution. The derivative of ESF yields LSF. Typically, in order to reduce the noise amplification at the edge points, a Hamming filter is applied. MTF is obtained by Fourier transforming the LSF data. ROI image was rotated in order to obtain the MTF at the other direction. In our method, fitting an analytical function to ESF data and applying Hamming window were not performed. Also, in our case, oversampling ratio depends on the region of interest and there is a cosine correction factor, which handles non standard edge angle effects. In Figure 3, we demonstrate our MTF extraction results for the sine wave star and slanted edge algorithms, in addition to theoretical input MTF. Our theoretical input and extraction results are in very good agreement.



**Figure 3.** MTF curves: i) ICS input data, ii) Sine wave star (SWS) Extracted MTF data, iii) Slanted edge extracted MTF from data using slightly modified (SM) ISO-12233 method.

The next step is to apply the image restoration algorithms. We use the following formula for image restoration, which is called Wiener and Helstrom filter.

$$W[f_x, f_y] = \frac{H^*[f_x, f_y]}{|H[f_x, f_y]|^2 + \frac{|N[f_x, f_y]|^2}{|F[f_x, f_y]|^2}} \quad (1)$$



**Figure 4.** (a) Sine wave star section, restored image, (b) MTF curves: i) ICS data, ii) Restored sine wave star (SWS) data, extracted MTF, iii) Restored slanted edge data using slightly modified ISO-12233 method, extracted MTF.

Here  $H^*[f_x, f_y]$  is the complex conjugate of OTF,  $|F[f_x, f_y]|^2$  is the original undegraded object power spectrum,  $|N[f_x, f_y]|^2$  is the power spectrum of the noise.

We investigated the problem of MTF extraction from restored images. In Figure 4 (a), restored images are shown. In Figure 4 (b), corresponding MTF extraction data were demonstrated.

### 3.1. Slanted angle dependence

MTF determination was performed with slanted edge patterns that allow super sampling of the edge spread function. In the absence of cosine factor correction, the edge angle affects the MTF extraction results. We investigated the variation in edge angle and compared our results for the 5°, 10°, 15°, and 20° cases. We only changed the edge angle and performed the analyses while keeping the other parameters the same. As the angle increased the results are shown in Figure 5. We observe that as the angle increased, the deviation becomes larger and even though the cosine correction factor reduces the deviation considerably [13]. The mean squared error is 0.0652%, 0.0724%, 0.0855%, 0.1155%, for the cases of 5°, 10°, 15°, and 20°, respectively. We see the error rate increases as the slanted edge angle increases. The extraction method becomes inadequate for slanted edge angle of 30° and more.

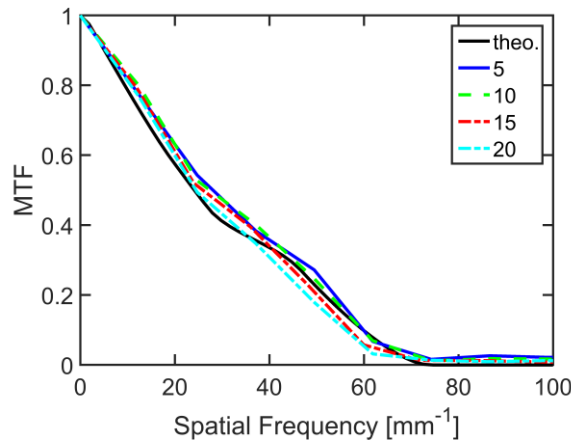
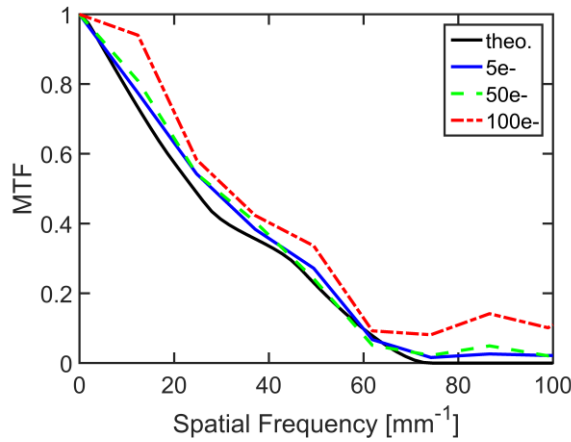


Figure 5. Effect of slanted edge angle on MTF extraction.

### 3.2. Noise effects and corresponding SNR

The SNR level could decrease due to the decrease in time delay integration steps or increase in dark noise. As the dark noise was increased from  $5e-$  to  $50e-$ ,  $100e-$ , we see the increase in noise level in the images. The extracted MTF becomes less accurate as the noise level increases. We did not apply any noise extraction algorithm to reduce the noise and observed increase of the deviations as shown in Figure 6. The mean squared error is 0.0652%, 0.1187%, 0.7790% for the  $5e-$ ,  $50e-$ ,  $100e-$  cases, respectively. The extraction method becomes inadequate when the noise becomes as high as  $150e-$ .



**Figure 6.** Effect of noise on MTF extraction.

### 3.3. Restoration Level

MTF values extracted from restored images are larger than degraded image case. The Wiener filter method was applied to the degraded images with image noise extraction utilizing median filter. For the case of lower performance restoration filters, we compared the MTF extraction results. The MTF extraction curves (not shown) indicated that when the ratio of power spectrum of the noise and object power spectrum was taken as a fixed constant number (knob parameter) reduced restoration performance was obtained. Other than good restoration case, the Wiener filter knob parameter was taken as fixed constant value. Such an implementation yielded much lower restoration performances and named here as bad restoration case. The PSNR values for the good and bad restoration cases were 23.90 and 15.43, respectively.

### 3.4. Selected Region of Interest

MTF extraction results depend on the selected region of interest. We change the ROI width and length simultaneously by keeping the shape as square. The decrease at the size of the ROI affects the extracted MTF values negatively. There is a particular interval for ROI size, which can be utilized in our method. The effect of ROI size becomes significant for the low spatial frequency data. In Figure 7, when ROI is  $2 \times 48$ ,  $2 \times 52$ ,  $2 \times 56$ ,  $2 \times 60$ ,  $2 \times 64$ , the MSE becomes 0.1732%, 0.0855%, 0.2360%, 0.0652%, 0.1279%, respectively. When ROI was changed within the limitations of the method, we do not see a particular increasing or decreasing trend. The benchmark ROI size was chosen as  $2 \times 60$  in our simulations.



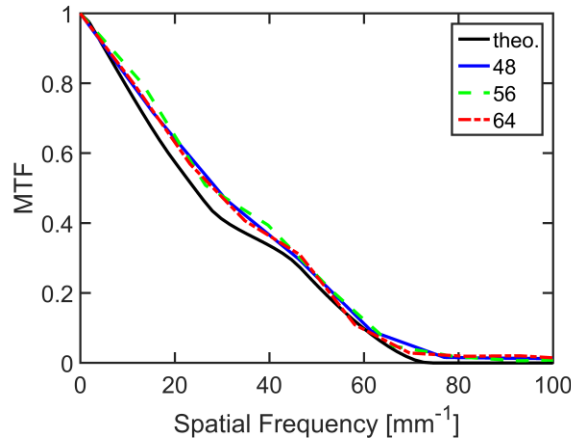


Figure 7. Effect of ROI size on MTF extraction.

### 3.5. Oversampling factor

In our analyses, oversampling factor was automatically selected and varies in accordance with the ROI data. The number of bins was almost always larger than the standard value of four. Therefore, we conclude that increasing the oversampling factor above four yields better results.

## 4. CONCLUSION

By comparing our MTF extraction results with the given theoretical input values, we showed that our extraction method is very accurate. A number of factors directly affect the extraction algorithms' performance are noise level, angle of the slanted edge, quality of restoration, selection of the region of interest, and oversampling number. The benchmark configuration yielded %0.013 mean squared error between the input MTF and slanted edge case. The mean squared error for the sine wave star pattern code was even less.

## ACKNOWLEDGEMENT

The author (KBA) has been supported by the Turkish Academy of Sciences (TÜBA), in the framework of the Young Scientist Award Program (TÜBA-GEBİP/2020).

The research study presented in this article was based on the views of the authors and does not reflect on the official policy of the TÜBİTAK, Ministry of Industry and Technology, or the Turkish government.

## REFERENCES

- [1] A. C. Bergstrom, D. W. Messinger, *Image quality and computer vision performance: assessing the effects of image distortions and modeling performance relationships using the general image quality equation*, Journal of Electronic Imaging **32**(2), 023018 (2023).

- [2] C. N. Reinhardt, Z. P. Ritsema, J. W. Clark, D. V. Opel, B. E. Young, and B. P. Teaney, "Enhanced physics-based modeling of the electrooptical/infrared observation chain," in *Proc. SPIE*, pp. 121060C, 2022.
- [3] S. Reichenbach, S. Park, and R. Narayanswamy, *Characterizing digital image acquisition devices*, Optical Engineering **30**(2), 170-177 (1991).
- [4] [R. Barakat, *Determination of the Optical Transfer Function Directly from the Edge Spread Function* Journal of the Optical Society of America **55**, 1217-1221 (1965).
- [5] B. Tatian, *Method for Obtaining the Transfer Function from the Edge Response Function*, Journal of the Optical Society of America **55**, 1014-1019 (1965).
- [6] J. K. M. Roland, "A study of slanted-edge MTF stability and repeatability," in *Image quality and system performance XII, SPIE*, 2015, pp. 181-189.
- [7] K. B. Alici, "Spatial resolution and modulation transfer function analyses for satellite imaging," in *11th Ankara International Aerospace Conference, 8-10 Sep. 2021, Ankara, 2021*, pp. AIAC-2021-136.
- [8] D. Williams, "Benchmarking of the ISO 12233 slanted-edge spatial frequency response plug-in," in *PICS, 1998*, pp. 133-136.
- [9] T. Newswander, D. W. Riesland, D. Miles, and L. Reinhart, "Slanted edge MTF testing for establishing focus alignment at infinite conjugate of space optical systems with gravity sag effects," in *Astronomical Optics: Design, Manufacture, and Test of Space and Ground Systems, 2017*, pp. 163-173.
- [10] K. Masaoka, *Edge-based modulation transfer function measurement method using a variable oversampling ratio*, Optics Express **29**, 37628-37638 (2021).
- [11] A. Tzannes and J. Mooney, *Measurement of the modulation transfer function of infrared cameras*, Optical Engineering **34**(6), 1808-1817 (1995).
- [12] T. Li, H. Feng, and Z. Xu, *A new analytical edge spread function fitting model for modulation transfer function measurement*, Chinese Optics Letters **9**, 031101 (2011).
- [13] K. Masaoka, M. Sugawara, and Y. Nojiri, "Multidirectional MTF measurement of digital image acquisition devices using a Siemens star," in *Digital Photography VI, 2010*, pp. 295-302.
- [14] C. R. Purcell, *Remote sensing image performance metrics: Comparing ground sample distance and the national imagery interpretability rating scale*. California State University, Long Beach: 2000.
- [15] C. Dickinson, *An evaluation of the current state of digital photography*. Rochester Institute of Technology, 1999.
- [16] M. Kameche and S. Benmostefa, *In-flight MTF stability assessment of ALSAT-2A satellite*, Advances in Space Research **58**, 117-130 (2016).
- [17] J. Li and Z. Liu, *Self-measurements of point-spread function for remote sensing optical imaging instruments*, IEEE Transactions on Instrumentation and Measurement **69**, 3679-3686 (2019).
- [18] cS. P. Sadoulet and B. Taylor, *Tilted edge for optical-transfer-function measurement*, U.S. Patent No. 7518712, (2009).

- [19] M. Etribeau and P. Magnan, "Pixel crosstalk and correlation with modulation transfer function of CMOS image sensor," in *Sensors and Camera Systems for Scientific and Industrial Applications VI, 2005*, pp. 98-108.
- [20] K. B. Alici, "Extraction of Modulation Transfer Function by using Simulated Satellite Images," in *10th International Conference on Recent Advances in Space Technologies (RAST), 2023*, pp. 1-5.
- [21] K. B. Alici, F. S. Oktem, O. Karci, A. S. Yilmaz, and O. Selimoglu, *Image Chain Simulation for Earth Observation Satellites*, IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing **12**, 4014-4023, (2019).
- [22] K. B. Alici, O. Karci, A. S. Yilmaz, C. Ozdemir, F. S. Oktem, and O. Selimoglu, "OTF analysis of a spaceborne CMOS imaging sensor," in *7th International Conference on Recent Advances in Space Technologies (RAST), 2017*, pp. 133-138.

**To Cite This Article:** K. B. Alici, *Modulation Transfer Function Extraction and Parametric Effects*, Journal of Aeronautics and Space Technologies **17**(Special Issue), 19-29 (2024).

#### **VITAE**

**Kamil B. Alici** received his B.Sc. and Ph.D. degrees in physics from Physics Department, Bilkent University, Türkiye in 2004 and 2010, respectively. He is currently working at TÜBİTAK Space Technologies Research Institute (TÜBİTAK UZAY) as Chief Scientist. Dr. ALICI received the Young Scientist Award of Turkish Academy of Sciences (TÜBA GEBİP/2020).