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Research Article

Angular Velocity Estimation for Nanosatellites Using Vector Measurements

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Abstract

In the absence of gyro sensors, the straightforward approach to estimate the angular velocity is using a dynamics-based attitude filter which can provide angular rate information together with the attitude of the satellite. However, this approach requires also the reference directions for the vector measurements. In this article, assuming that the reference directions are not known, two different algorithms for angular velocity estimation, using only the body vector measurements are given and compared. They both rely on pre-filtering the noisy data, specifically for the magnetometer measurements. The first algorithm estimates the Residual Magnetic Moment (RMM) terms along with the angular velocity vector. The second algorithm does not use the spacecraft dynamics and model the angular accelerations as a first-order Markov process. Demonstrations show the first algorithm can provide an accuracy up to $\sim 0.035^{o}/s$ for angular velocity estimates and accurately estimate the RMM terms, whereas the second algorithm is a computationally efficient alternate that can be used flexibly at any mission phase.

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Nano Uydular için Vektör Ölçümleri Kullanılarak Açısal Hız Kestirimi

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Öz

Dönüölçerlerin yokluğunda açısal hız kestirimi için kullanılabilecek en bilinen yöntemlerden biri dinamik tabanlı bir filtre ile hem açısal hızların hem de uydunun yöneliminin kestirimidir. Fakat bu yaklaşım vektör ölçümleri için referans yönelimleri de gerektirir. Bu makalede referans vektör yönlerinin bilinmediği varsayılarak, sadece gövde eksen takımında ölçülmüş vektörleri kullanan iki farklı açısal hız kestirim yöntemi sunulmuş ve karşılaştırılmıştır. Her iki yöntem de özellikle manyetometreler için gürültülü verilerin öncül bir filtreden geçirilmesine dayanmaktadır. İlk algoritma açısal hızların yanısıra Artık Manyetik Moment (AMM) terimlerini de kestirmektedir. İkinci algoritma uydu dinamiklerini kullanmamakta, açısal ivmeyi birinci mertebeden bir Markov süreci şeklinde modellemektedir. Benzetimler göstermektedir ki ilk algoritma $\sim 0.035^{o}/s$ mertebesinde bir açısal hız kestirim doğruluğuna erişebilmekte ve AMM terimlerini doğru olarak kestirebilmekte iken, ikinci algoritma farklı görev safhalarında da kullanılabilecek hesaplama açısından etkin bir alternatif sunmaktadır.

1. INTRODUCTION

Angular velocity of a spacecraft is a required information for all missions to control and stabilize the attitude. If the spacecraft is equipped with a gyroscope, this information is obtained from the gyro readings. However, for many nanosatellite missions, the spacecraft does not have a gyro sensor at all, due to the budget constraints. Even, if a gyro can be used, having back-up algorithms, which can provide angular velocity information without requiring gyros, can be advantageous since this is an essential information to control and stabilize the spacecraft [1]. Thus, angular velocity estimation using the measurements of other attitude sensors becomes a required task for many missions [2-5].

Angular velocity estimation methods for a satellite can be categorized mainly in three, depending on the sensors (or available information) used for estimation and the structure of the estimation algorithm. First and most straightforward approach to angular velocity estimation is using the known attitude information [3, 6-8]. The attitude information might be obtained directly from a star tracker or estimated by using the vector measurements (and the associated reference direction) in an algorithm such as QUEST [9]. Once the attitude is known at sequential instants, angular velocity information can be inferred either by simply differentiating the attitude [3] or using quaternions as measurements in an attitude filter (see Block I in Fig.1.) The filter should be a dynamics-based filter [10] in this case since there is no gyro onboard.

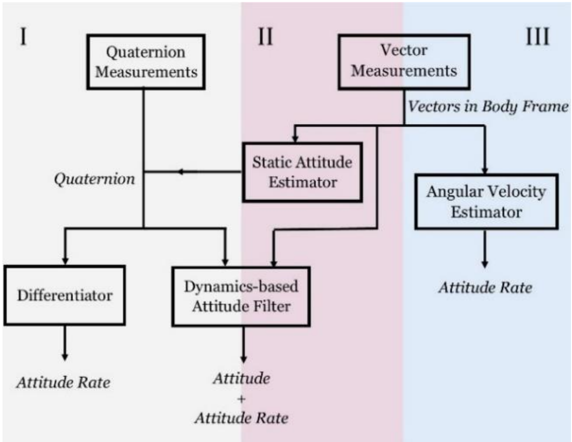

Figure 1. Classification of angular velocity estimation methods for a satellite.

In second approach (Block II in Fig.1) the vector measurements in satellite body frame, which are obtained from sensors such as magnetometers, are used in a dynamics-based attitude filter to estimate both the attitude and attitude rate of the satellite [1, 11-13]. These methods require also the reference directions associated with the measurements in the body frame (e.g., magnetic field direction vector in the Earth centered inertial frame corresponding to the magnetometer measurements in the body frame). These reference direction vectors can be modelled (calculated) using the time and satellite position information. The nonlinear observer given in [4] might be also considered in this category, since it requires the reference direction vectors.

Third and last approach (Block III in Fig.1) for angular velocity estimation, uses only the measured vectors in the body frame and does not require any other information such as the modelled reference directions. This is advantageous especially in lost-in-space conditions when the satellite's exact position is not known and the angular velocity must be estimated using solely the vector measurements. Several different methods are available in literature which are falling into this category: Iterative algorithms [14], static estimators (so called Wahba problem based methods) [2] and Kalman rate estimators [15, 16]. Kalman rate estimators among these methods are most advantageous, since they can be used even with a single vector measurement, e.g., the magnetometers.

In this article, we propose two approaches for spacecraft angular velocity estimation, which are loosely based on the idea of Kalman rate estimator given in [15, 16]. Both approaches rely on pre-filtering the noisy sensor data to calculate the derivatives of the vector measurements at the first stage of the algorithm. A separate Kalman filter (KF) as in [17] is used for this purpose because of its simplicity and computational efficiency fitting to the nanosatellite requirements. In the second-stage of the algorithm, the estimated derivatives are used as measurements to estimate the angular velocity of the spacecraft. The first approach estimates also the residual magnetic moment (RMM) vector and compensates in the satellite dynamics, considering that the magnetic disturbance is the dominant disturbance for a nanosatellite [18, 19]. The later stage of the algorithm is formulated as an Extended KF (EKF). In the second approach, the rate estimator is designed without using the spacecraft dynamics model. Instead, the angular acceleration is modeled as a zero-mean stochastic process with exponential autocorrelation function. This approach saves us from using uncertain dynamics model that is under effect of also the other disturbance torques (e.g., gravity gradient) which cannot be accounted for unless the attitude is known. Besides the angular velocity can be estimated using a linear KF algorithm.

Both the proposed approaches can work without the spacecraft position and attitude information. Thus, they are suitable for de-tumbling and initial acquisition phases. Even a single vector measurement is enough to estimate the angular velocity. Application examples in the article will be based on using the magnetometer and Sun sensor measurements. Yet the algorithms can be extended such that multiple vector measurements are processed simultaneously. As the main contributions, we show that: 1) a two-stage algorithm is a computationally efficient way for both calculating the vector derivatives and estimating the angular velocity vector; 2) the RMM vector can be estimated along with the angular velocities and compensated in the dynamics by using only the magnetometer measurements; 3) a coarse angular velocity estimate within the bounds of $1^{\circ}/\mathrm{s}\ (3\sigma)$ is possible without using the spacecraft dynamics at all.

The rest of the article is organized as follows: In Section 2, the mathematical model for the spacecraft dynamics and the method to infer angular velocity from vector measurements are presented. In Section 3, the KF algorithms including the pre-filter for derivative estimation are given. The EKF for both angular velocity and RMM estimation and the KF for angular velocity estimation without requiring the spacecraft dynamics are explained in detail. Section 4 demonstrates the proposed algorithms for a nanosatellite using the simulated data. Concluding remarks are given in the Section 5.

2. PRELIMINARIES

2.1. Spacecraft Dynamics

Euler's rotational equation for spacecraft dynamics can be given as

$$\dot{\boldsymbol{\omega}} = J^{-1}\left(\mathbf{L}_d - \boldsymbol{\omega}\times\left(J\boldsymbol{\omega}\right)\right) \quad (1)$$

Here $\boldsymbol{\omega}$ is the vector defining the angular velocity of spacecraft body frame with respect to the inertial frame and resolved in the body frame. It would be shown as $\boldsymbol{\omega}_{ib}^{b}$ with proper subscripts and superscripts, which are omitted for simplicity here. J is spacecraft moment of inertia tensor and $\mathbf{L}_d$ is the vector of disturbance torques acting on the spacecraft in the body frame.

In general, there are four main disturbance torques acting on a spacecraft and which might be accounted for: The gravity gradient torque, solar pressure torque, aerodynamic torque and the magnetic disturbance torque. For a small spacecraft in Low Earth Orbit (LEO), the dominant one amongst them is the magnetic disturbance [18], [19] which can be modelled as

$$\mathbf{L}_d = [\mathbf{M}\times]\mathbf{B_b} \quad (2)$$

Here $\mathbf{M}$ is the RMM vector and $\mathbf{B_b}$ is the magnetic field vector, both in the body frame. $\mathbf{B_b}$ vector can be directly obtained from the magnetometer readings. On the other hand, for compensating the magnetic disturbance, whenever the dynamics equation is used, the RMM vector must be estimated [10]. In this article, first of the proposed techniques estimates the RMM vector along with the angular velocity vector. Since the second technique does not use the spacecraft dynamics, RMM information is not needed in the estimator architecture.

In Eq. 2 $[\boldsymbol{v}\times]$ denotes the cross-product matrix which is given as

$$[\boldsymbol{v}_\times] = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \quad (3)$$

where, $\boldsymbol{v} = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^T$. The actual angular velocities for the spacecraft are modelled by accounting for also the gravity gradient torque in this study. The torque can be calculated by

$$\mathbf{L}_{gg} = \frac{3\mu}{|\mathbf{r}|^3}\mathbf{n}\times(J\mathbf{n}) \quad (4)$$

Here, $\mathbf{n}$ is the nadir pointing direction in spacecraft body frame, which can be obtained as $\mathbf{n} = A\mathbf{r}_i$ if the spacecraft's attitude (A) and the position in inertial frame ($\mathbf{r}_i$) are known. μ is the gravitational parameter for Earth. In the designed angular velocity estimator, the gravity gradient torque cannot be accounted for, unless the attitude and position of the spacecraft are known.

2.2. Angular Velocity from Vector Measurements

The observation model for the proposed angular velocity estimation algorithms is based on the time derivative of the measured vectors. Let us follow the derivation process for the magnetometer measurements. The process can be easily generalized for any other vector measurement.

First of all, the magnetometer measurement vector in body frame, $\mathbf{B_b}$, and the reference direction vector in the inertial frame, $\mathbf{B_i}$, can be associated via the attitude matrix as

$$\mathbf{B_b} = A\mathbf{B_i} \quad (5)$$

Taking derivative of both side and using the fact

$$\dot{A} = -[\boldsymbol{\omega}\times]A \quad (6)$$

gives

$$\dot{\mathbf{B}}_\mathbf{b} = A\dot{\mathbf{B}}_\mathbf{i} - [\boldsymbol{\omega}\times]A\mathbf{B_i} \quad (7)$$

For most orbits, $\dot{\mathbf{B}}_\mathbf{i} \approx \mathbf{0}$ assumption can be made, since any change in the reference direction vector due to the change in the spacecraft's position and Earth's rotation is negligible within short sampling periods of magnetometer measurements. Thus, we are left with

$$\dot{\mathbf{B}}_\mathbf{b} = -[\boldsymbol{\omega}\times]\mathbf{B_b} = [\mathbf{B_b}\times]\boldsymbol{\omega} \quad (8)$$

which is indeed the attitude and reference vector independent observation equation to be used in the angular velocity estimation algorithms. Only required information is the vector measurements in the body frame and their derivatives.

3. ANGULAR VELOCITY ESTIMATION

3.1. Pre-filter: KF for Derivative Estimation

Eq.8 showed that we need to calculate the derivatives for the body frame vector measurements to build the observation model in the estimator. The numerical differentiation (e.g. forward difference) might give inaccurate results for such calculation, specifically for the magnetometer measurements, due to the high sensor noise. Thus, a more robust approach might be considered.

There are various algorithms in literature that can be used for differentiating the noisy signals [20]. However, the algorithm used onboard a nanosatellite must be computationally light as much as being accurate. Considering this requirement, we use a linear KF algorithm, which incorporates a third-order Markov process to model the magnetic field [17]. The model is given by

$$\frac{d^3\mathbf{B_b}}{dt^3} = \boldsymbol{\upsilon} \tag{9}$$

where $\boldsymbol{\upsilon}$ is white Gaussian process noise. The third-order Markov model, which is used to estimate the first and second derivatives of the vector as well as the vector itself, is given by

$$\dot{x} = \frac{d}{dt}\begin{bmatrix}\mathbf{B_b}\\ \dot{\mathbf{B}}_\mathbf{b}\\ \ddot{\mathbf{B}}_\mathbf{b}\end{bmatrix} = \begin{bmatrix}\dot{\mathbf{B}}_\mathbf{b}\\ \ddot{\mathbf{B}}_\mathbf{b}\\ \mathbf{0}_{3\times1}\end{bmatrix} \tag{10}$$

Here $\mathbf{0}_{3\times1}$ is 3×1 vector of zeros. The process and measurement matrices for the KF are

$$F = \begin{bmatrix}0_3 & I_3 & 0_3\\ 0_3 & 0_3 & I_3\\ 0_3 & 0_3 & 0_3\end{bmatrix} \tag{11}$$

$$H = [I_3 \quad 0_3 \quad 0_3] \tag{12}$$

where I_3 is 3×3 identity and 0_3 is 3×3 null matrix. Figs. 2 and 3 shows the results when the introduced KF is used for estimating the magnetic field derivatives in the body frame. The KF estimation results are compared with the actual derivatives, which are simulated using the noise free vectors in the body frame. Fig.3 also includes the errors for derivatives when they are calculated with forward-difference numerical differentiation algorithm. Note that all the results are given for normalized vector measurements. The magnetometer measurements are simulated with a white noise of standard deviation of $\sigma_m = 300\text{nT}$. There are no other errors (e.g. bias) affecting the measurements.

Simulations are done for a nanosatellite on a Sun-synchronous circular orbit with an orbital period of approximately 98 min. The tensor of inertia matrix is as follows $J = \text{diag}(0.037 \quad 0.037 \quad 0.01)\,\text{kgm}^2$. The dynamics of the satellite is under the effects of the gravity gradient torque and the magnetic disturbance torque. The assumed RMM value is $\mathbf{M} = [-0.09 \quad 0.001 \quad 0.11]^T\ \text{Am}^2$ and it is matching with the experienced level of RMM for actual nanosatellite missions [10]. There is no control torque acting on the satellite.

Both Figs. 2 and 3 clearly show that the used KF algorithm can accurately estimate the derivatives for magnetometer measurements. Specifically, Fig. 3 signifies that the KF approach can provide up to 5 times more accurate derivative estimates compared to the numerical differentiation. Derivative estimation accuracy for the KF algorithm for magnetometer measurements, which can be read from the plotted $\pm3\sigma$ bounds, is about $0.0105\ \text{s}^{-1}$.

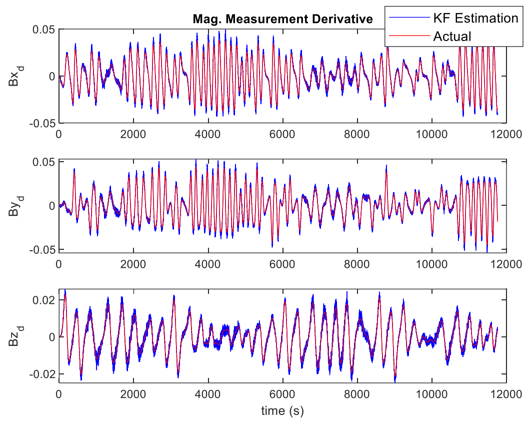

Figure 2. Derivatives for magnetometer measurement vector in body frame.

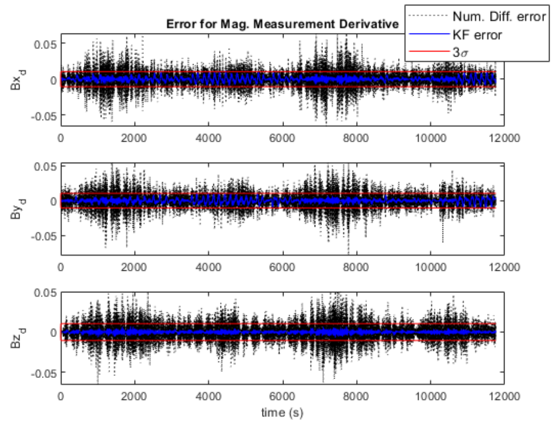

Figure 3. Errors for magnetometer measurement derivative estimation.

3.2. EKF Algorithm for Angular Velocity and RMM Estimation

Once the derivatives for the vector measurements in the body frame are obtained, they can be used in a filter designed in the EKF framework to estimate the angular velocities and the RMM vector (e.g. $x = [\boldsymbol{\omega} \quad \mathbf{M}]^T$). The EKF uses discretized version of Eq.1 as one of the system equations. For the other system equation, the RMM vector is modelled as constant

$$\frac{d\mathbf{M}}{dt} = \mathbf{0}_{3\times1} \tag{13}$$

The system matrix for the filter is built of the partial derivatives as

$$F = \begin{bmatrix} J^{-1}\left(\left[J\boldsymbol{\omega}\times\right]-\left[\boldsymbol{\omega}\times\right]J\right) & -J^{-1}\left[\mathbf{B_b}\times\right] \\ 0_3 & 0_3 \end{bmatrix} \tag{14}$$

Furthermore, in line with Eq.8, which is the observation equation relating the angular velocity vector with the body vector derivatives used as measurement, the measurement matrix can be given as

$$H = \begin{bmatrix} \left[\mathbf{B_b}\times\right] & 0_3 \end{bmatrix} \tag{15}$$

Note that Eq.15 is for an EKF using only the magnetometer measurements for angular velocity and RMM estimation. In case other vector measurements are also used, additional rows should be added in the H matrix, e.g. $\begin{bmatrix} \left[\mathbf{S_b}\times\right] & 0_3 \end{bmatrix}$ for Sun sensor measurements.

3.3. KF for Angular Velocity Estimation Using Markov Process Model

In this second algorithm for angular velocity estimation, the spacecraft dynamics model (Eq.1) is replaced with a system model that is modelling the angular acceleration as a zero-mean stochastic process with exponential autocorrelation function [12]:

$$\ddot{\boldsymbol{\omega}} = -\Psi\,\dot{\boldsymbol{\omega}} + \boldsymbol{\nu} \tag{16}$$

Here

$$\Psi = \mathrm{diag}\begin{pmatrix} \frac{1}{\tau_1} & \frac{1}{\tau_2} & \frac{1}{\tau_3} \end{pmatrix} \tag{17}$$

and $\boldsymbol{\nu}$ is zero-mean Gaussian white noise with covariance of $\mathrm{E}\left(\boldsymbol{\nu}_i\boldsymbol{\nu}_j^T\right) = 2\Psi\boldsymbol{\Sigma}^2$. To determine the noise variance of $\boldsymbol{\Sigma}$, the angular acceleration probabilistic model, whose details are provided in [12] can be used. τ_i for $i = 1,2,3$ in Eq.17 are the acceleration decorrelation times in each body axis.

There are two main reasons to replace the spacecraft dynamics model in this second algorithm:

1) The spacecraft dynamics model can be highly uncertain due to the unmodeled disturbance torques and errors in the dynamics parameters (such as the inertia tensors). Unless the attitude and position of the spacecraft is known, we cannot model the disturbance torques and attenuate the uncertainty.

2) Computational load, which is one of the primary concerns for nanosatellites, can be reduced with a simpler estimation algorithm structured as a linear KF.

As a result, the KF estimates also the angular accelerations along with the angular velocities. Thus, the state vector is $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{\omega} & \dot{\boldsymbol{\omega}} \end{bmatrix}^T$. The measurement model for this second approach is exactly the same with the model in the first algorithm and Eq.15 is used as measurement matrix. As to the state transition matrix, it becomes

$$F = \begin{bmatrix} 0_3 & I_3 \\ 0_3 & -\Psi I_3 \end{bmatrix} \tag{18}$$

4. NUMERICAL RESULTS AND DISCUSSION

In this section the proposed approaches for nanosatellite angular velocity estimation are tested using the simulated data and compared. Demonstrations are for the nanosatellite whose modelling details are given in Section 3.1. Orbit parameters, sensor specifications and assumed dynamic parameters are all same with those previously presented. The satellite is tumbling uncontrolled and under effects of the gravity gradient and the magnetic disturbance torques. Initial angular velocity vector is $\boldsymbol{\omega} = \begin{bmatrix} 0 & -0.6 & 0 \end{bmatrix}^T$ deg/s. Results are examined individually for cases with only magnetometer measurements, and magnetometer and Sun sensor measurements. Standard deviation for Sun sensor noise is $\sigma_s = 0.1^{\circ}/\mathrm{s}$. In the last subsection, we discuss briefly the algorithms' accuracy in the presence of sensor calibration errors.

4.1. Case 1: Estimation using only Magnetometer Measurements

Figs. 4 and 5 present the angular velocity estimation results when the EKF algorithm is used. For reference and noting the actual angular velocity profile throughout the estimation process, Fig.4 gives the EKF estimation results together with the actual values. Fig.5 presents the estimation errors and the associated $\pm 3\sigma$ bounds.

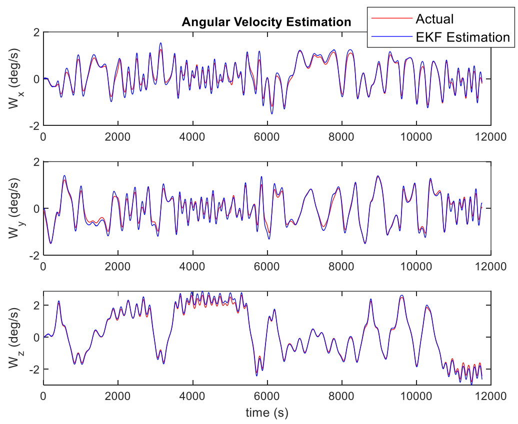

Figure 4. Angular velocity estimation results for the EKF when only magnetometer measurements are used.

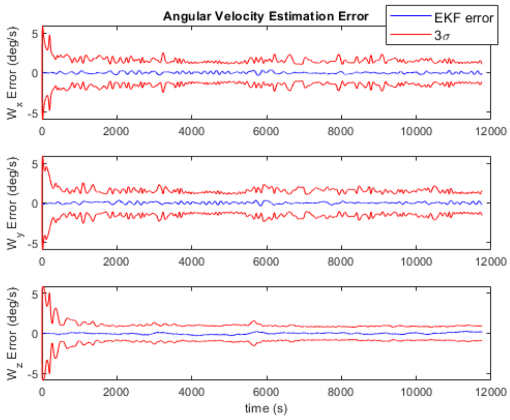

Figure 5. Angular velocity estimation errors for the EKF when only magnetometer measurements are used.

When only the magnetometer measurements are used for angular velocity estimation the accuracy is limited due to both the sensor errors and the inertial rotation of the Earth's magnetic field, which is neglected in Eq.8. Moreover, of course the gravity gradient disturbance torque, which cannot be accounted in the filter system equation, is contributing to the estimation error. Nonetheless, the results specifically in Fig.5, and the root mean square error (RMSE) values in Table 1 (presented at the end in Section 4.2 to sum up and compare the results for individual cases) show that the angular velocities can be estimated with an accuracy up to $0.12^{\circ}/\mathrm{s}\ (1\sigma)$ using only the magnetometer measurements. Indeed, estimating also the RMM terms and compensating in the dynamics is an important contributing factor for achieving such accuracy level. Fig.6 shows that the EKF can accurately estimate the RMM terms along with the angular velocities despite using only the magnetometer measurements.

Essentially the accurate RMM estimation with the proposed EKF algorithm has benefits more than just increasing the angular velocity estimation accuracy. Since the RMM terms are now known, these estimates can be used in the main attitude filter as well, once the position information for the spacecraft, thus the reference vector directions in the inertial frame are available.

Fig.7 gives the angular velocity estimation errors when KF with the first-order Markov process model for angular acceleration is run. Clearly, the KF estimates are almost 3 times inaccurate than those of the EKF (see also Table 1). Especially, the body Z axis angular velocity estimation accuracy is poor due to the experienced disturbance torque along this axis with minimum moment of inertia. The Markov process model falls behind at representing the actual angular acceleration profile specifically for this axis.

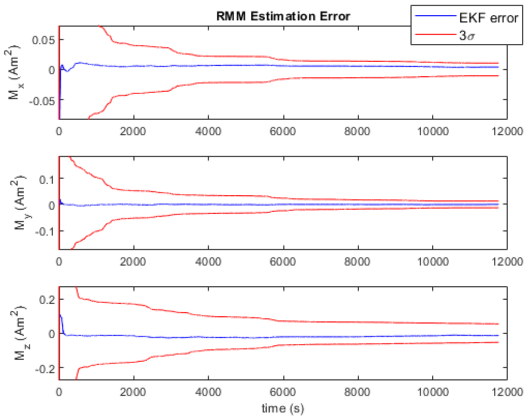

Figure 6. RMM estimation errors for the EKF when only magnetometer measurements are used.

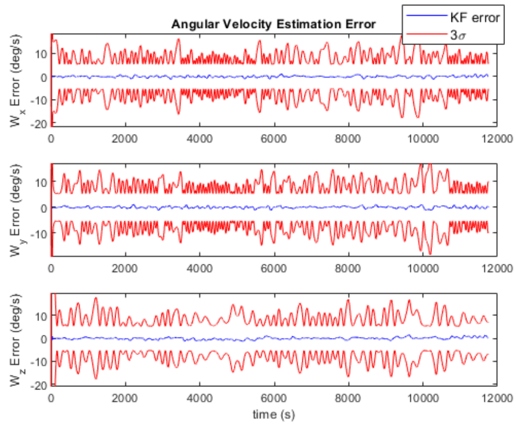

Figure 7. Angular velocity estimation results for the KF when only magnetometer measurements are used.

One main advantage of the KF algorithm for angular velocity estimation is the simplicity. Numerical evaluations by comparing the run times of the algorithms in MATLAB environment show that the computational load decreases almost 40% when the KF algorithms is used. Thus, this algorithm can be useful when a backup solution for the angular velocity estimation is needed. Using only the magnetometer measurements, coarse, but continuous angular velocity estimates can be obtained. Since it is computationally non-demanding, as well as the initial acquisition phase, such algorithm can be run for the whole mission duration for fault-checking the angular velocity estimates by the main attitude filter.

4.2. Case 2: Estimation Using Also the Sun Sensor Measurements

Sun sensors is one of commonly used attitude sensors for nanosatellites along with the magnetometers. Due to this fact, examining the accuracy of the proposed algorithms with these two sensors might be insightful. Furthermore, Sun direction changes only $\sim 1^{\circ}/\mathrm{day}$. Thus, assuming that the Sun reference direction vector in the inertial frame is not changing within short measurement sampling periods (e.g. $\dot{\mathbf{S}}_{\mathbf{i}} \approx \mathbf{0}$) does not introduces errors to the estimated angular rates as much as same assumption with the magnetic field reference direction does [16].

Figs. 8 and 9 give angular velocity estimation errors for EKF and KF, successively. The shadow regions indicate the eclipses when the Sun sensor measurements are not available. These results and also the RMSE values presented in Table 1 show that incorporating the Sun sensor measurements in addition increases the accuracy to a level more than $0.08^{\circ}/s\ (1\sigma)$ for the EKF and $0.28^{\circ}/s\ (1\sigma)$ for the KF. Note that the RMSE values in Table 1 are calculated for the whole estimation period including the eclipses, when only magnetometer measurements are used. If eclipse periods are excluded and the RMSE values for the filters are calculated for only sunlit periods, the EKF error reduces to $\sim 0.035^{\circ}/s$ and the KF error reduces to $\sim 0.2^{\circ}/s$ in all three axes. In particular the EKF results are in agreement with the achived accuracy level in literature. For example in [16] it is noted that the RMSE is $\sim 0.02^{\circ}/s$ when only Sun sensor measurements are used in a filtering algorithm with a similar structure. Having said that neither the gravity gradient disturbance nor the magnetic disturbance are accounted for in [16]. Hence, our results are comparable when we consider that the given EKF is estimating also the RMM terms and the filter dynamics model is not completely accurate.

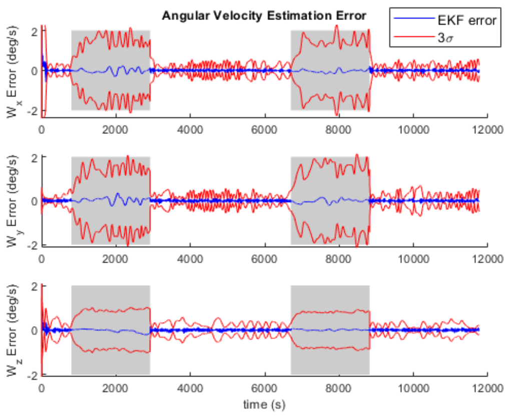

Figure 8. Angular velocity estimation results for the EKF when magnetometer and Sun sensor measurements are used. The shaded areas in the graphs indicate eclipse period.

As to the KF algorithm, incorporating the Sun sensor measurements, improves the angular velocity estimation accuracy specifically in Z axis since now the filter can rely more on the measurements rather than the first-order Markov process model. Additional Sun sensor measurements do not change the computational load of the algorithms significantly and the KF is still far less demanding.

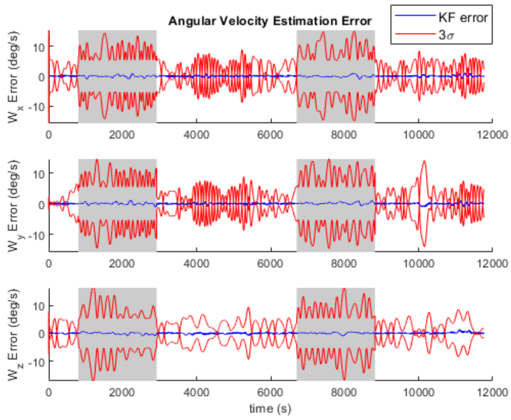

Figure 9. Angular velocity estimation results for the KF when magnetometer and Sun sensor measurements are used.

Table 1. RMSE Values for the proposed angular velocity estimation algorithms for different sets of available measurements.

	EKF		KF	
	Mag	Mag+Sun	Mag	Mag+Sun
ω_x ($^{\circ}$/s)	0.1227	0.0790	0.3607	0.2726
ω_y ($^{\circ}$/s)	0.1202	0.0687	0.3633	0.2820
ω_z ($^{\circ}$/s)	0.1101	0.0491	0.4714	0.2812

4.3. Case 3: Estimation in the Presence of the Sensor Errors

At the initial acquisition phase, when the proposed algorithms are most useful, it is very likely that the attitude sensors will be uncalibrated. This is specifically a problem when the magnetometers measurements are used. It is known that, small satellite magnetometers require in orbit calibration [21]. Unless, the magnetic reference direction vector is available there is no method to calibrate the magnetometers. Thus, running the angular velocity estimation algorithms with uncalibrated measurements becomes inevitable.

Table 2 briefly evaluates the accuracy of the proposed algorithms when the magnetometer measurements are fully distorted by constant bias, scaling and nonorthogonality errors. Only magnetometer measurements are used in the algorithms. The errors are assumed as

$$\mathbf{b_b} = \begin{bmatrix} 5000 \\ 3000 \\ 4000 \end{bmatrix} \text{nT} \qquad D_b = \begin{bmatrix} 0.1 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.1 \end{bmatrix} \tag{19}$$

Here $\mathbf{b_b}$ is the bias vector and D_b is the scaling and nonorthogonality matrix. Reader may refer to [21] for the details of these parameters and a complete measurement model for magnetometers when all these errors are accounted for.

RMSE values presented in Table 2 shows that magnetometer errors deteriorate the accuracy of both algorithms. Yet they have higher impact on the EKF's accuracy. This is mostly due to the poor RMM estimation and its reflection on the dynamics. As presented in Eq.2 the magnetic disturbance torque is a function of the magnetic field in the body frame. In the absence of any attitude and reference magnetic field information we have to use the magnetometer measurements. Hence the distorted measurements are directly affecting the RMM estimation and the dynamics equation we use in the EKF. On the other hand, as the results show, the KF, which does not use the spacecraft dynamics, is not that much affected by the magnetometer errors and achieves even more accurate results for angular velocities in body X and Y directions. Thus, in the presence of magnetometer errors, using the KF algorithm can be suggested, especially if the angular velocity is estimated using only the magnetometer measurements.

Table 2. RMSE Values for the proposed angular velocity estimation algorithm when uncalibrated magnetometer measurements are used.

	EKF	KF
$\omega_x\,(^\circ/s)$	0.4529	0.4191
$\omega_y\,(^\circ/s)$	0.4365	0.4106
$\omega_z\,(^\circ/s)$	0.4268	0.5094

5. CONCLUSION

In this article, we presented two different approaches for estimation of the body angular velocities for a nanosatellite using the coarse vector measurements. In the absence of gyro sensors, if the reference direction vectors for the measurements are also not known, only the body vector measurements are left to infer the spacecraft angular velocity estimates. In this article, the proposed algorithms are built on these assumptions. They both rely on pre-filtering the noisy data, specifically for the magnetometer measurements. The first algorithm, structured as an Extended Kalman Filter (EKF), estimates also the residual magnetic moment (RMM) terms along with the angular velocity vector to compensate the magnetic disturbance torque in the spacecraft dynamics. The second algorithm does not use the spacecraft dynamics at all and instead uses the first order Markov process model for the angular accelerations. It is simpler and in form of a KF algorithm. Algorithm evaluation for simulated data in different cases show that each angular velocity estimation algorithm has its own advantages. The first algorithm, the EKF, which also estimates the RMM terms, can provide accurate angular velocity estimates, especially when the magnetometer and Sun sensor measurements are used jointly. Conversely, the KF with first-order Markov process angular acceleration model, is computationally much efficient and more robust to any sensor calibration error. Depending on the desired task and the available computational resources, both algorithms can be used for de-tumbling the gyroless nanosatellites at the initial acquisition phase and also as back-up algorithms for the whole mission duration. They can provide angular velocity information when the position and timing information are lost. Furthermore, their estimates can be used for fault-checking the angular velocity estimates by the main attitude filter.

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VITAE

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