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Research Article

Plasma Thruster with Diffusion Control Using Real-Time Simulations with Observer Structure and Ionising Radiation Caused by Electrodynamic Effects in the Accelerator Chamber

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Abstract

The script in this paper describes the use of propellant for plasma thrusters based on electrostatic accelerator. The generation and control of plasma is essential for the ion thrusters. The effects of generated plasma may be used for simulation with heavy ions also. Since of used neutralisation by most types the ion current with the electron current unites to one neutral mass current with very high speeds. The types of achieved energies about 10 keV by first ionisation are state of the art and about 15 keV of second ionisation are rare types also. The cathode has to be able to withstand this energy permanently because some of the ions hit with its grid. The beam of them differs from typical alpha radiation. The ionisation chamber with the accelerator generate high speed of the ions, which produce the ionising and electromagnetic radiation due acceleration. This process may be used for the radiation tests also.

With the proposed mathematical model it is possible to calculate the spatial distribution of the electromagnetic fields as well the spatial temperature distribution in real-time. For this purpose are radiation hardened electronic systems with microcontroller, operational amplifiers and FPGA required for control with an observer structure working with parallel real-time simulation based on state-space representation with an additional model for diffusion effects. With an observer system, the disturbances may be determined. This provides the estimation of the more precise systems states for the control and additionally the estimation of the disturbances that arise due to radiation effects.

The results have also shown that an observer system can be developed specifically for the real-time calculation of the radiation effects only. The possibilities are related to all discrete solutions of differential equation systems, linear time invariant, the partial differential equations and the high performance digital mathematic operations. The paper gives also an overview of the diffusion effects and their mechanisms. The consideration is made by proposed extension of the Fick's laws and Maxwell equations; the introduction of new vector sizes is performed also. The analytical results were good enough for calculation of fields with diffusion. Using the advanced control method of thrust, a control structure for the electric test system could be created.

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1. INTRODUCTION

With expanding space industry interest for new technologies becomes bigger. One of the main aims is the development and completion flights up to the moon and farther. With ability to carry out secure the spacecraft with people also or other biological systems beyond the LEO, extended thruster systems and simulation-environment-systems on-board are necessary. They have to withstand the GCR (Galactic Cosmic Radiation) and the solar radiation without protection of the earth's magnetic field. The fast spacecraft intended for future flights are supposed to be designed according to the modus for the upper layer of the atmosphere, LEO and beyond. For the spacecraft systems not only fault-tolerant, radiation hardening, safety systems are needed. The latter systems increase to a large extent the stable calculations. This refers primarily to simulation systems that are comparable to systems used in present, whereby the user can rely on the results. Especially in the area of deep space missions the decentralisation without communication, with disturbances and unforeseen impacts of the spacecraft becomes more important. This assumes the fully autonomous system, which has the equipment to deal with any most difficult challenges in far places of space.

The simulations can contain a software-based assistant with a designed control structure and weak AI that can give a help by the complex evaluation procedure. For reaching full longer autonomy there are, in general, problems in main categories to solve; the electromagnetic propulsion (plasma thruster) with design of robust propulsion systems and advanced control. For this purpose the avionics includes a secure navigation, radiation hardened integrated circuits and secure communication. The spacecraft may include radiation detections systems, the observation of on-board systems, calculation engine for temperature distribution in real time to solve plasma and diffusion effects and computing other problems. Very important is a design and solution of power supply of thrusters and spacecraft systems. These categories require the highest assurance level.

In terms of initially mentioned motivation, the study is limited only to the accelerator, calculation of the propulsion and real-time simulation environment for complex observer structures. The subsystems are considered and simplifications are introduced in order to maximise the computing speed of simulations and other calculations. The real-time simulations with the robust designed electronics allow the user to have completely new perspectives of both, safe flights and real-time calculations of systems on-orbit.

Electronic parts can take damage caused by impacts with charged particle flux in space. In order to be able to react to these processes, it must be calculated and detected within a shorter time that ionising radiation is present. For this purpose, all available sensors may be used for the most realistic evaluation of the diffusion and ionising effects of the test system. For this purpose, the temperature sensor may be used for the evaluation of the dynamic temperature increases from which a measure of the ionisation processes and thus radiation may be detected. In addition, the charge detecting elements can be used to record pulse, vibrations and oscillations to absorb impacts of particle flux. At beginning the sensors can be used to calibrate the spatial distributions also. By measured value of size and known location of the sensors, the entire distribution can be calculated retroactively or more accurately. With the formation the type of ionisation and the direct effect to the systems and thus possible prevent processes can be activated up to the shutdown.

The analysis shows possibilities to perform more qualitative and faster simulations with calculations independent of space-systems and radiation environment also. The consideration of the electrodynamic influence on the signals was absolutely necessary, especially at higher sampling frequencies (1 kHz to 100 MHz). Within the scope of this work, a mathematical model of the consideration of the electromagnetic diffusion effects of arbitrary electric current and electric voltage "waveforms" has been developed. The main advantage of the proposed calculation of diffusion is the real-time capability, which is not possible with the FEM programs available today. Atomisation and sprays are descripted in [1]. Ultra-low emissions and properties are descripted in [2]. Subordinate Systems may be used for calculations and observations also and communicate with the main system. The communication between the systems, especially belonged to a control system shall have a fault-tolerant function. The possible solution may be found in [3].

2. ELECTROMAGNETIC ION THRUSTER

The main purpose of this paper is the description of an accelerator system capable of accelerating ions. This causes; both the electrodynamic and hydrodynamic processes and play a significant role in both; generation and acceleration. To reach LEO, engines with direct combustion are still used. It is also known that the plasma-powered thrusters cannot still achieve the required net thrust for direct launches. The control of chemical reactions and demand for energy and fuel are still challenging issues.

However, the plasma thrusters are very efficient outside the gravitational influence of the earth or other planets. They can accelerate the spacecraft to high speeds. For rotational dynamics the plasma thrusters may be used also. However the more efficient in this case is the application of reaction wheels including magnetic bearing. The literature is given in [4] and [5]. Example to active bearing can be found in [6], but the passed idea for passive bearing using a resonant circuit is given in [7]. More to parallel control is described in [8]. For rotation a specific angular momentum, referenced to a specific impulse, may be defined. The momentum may reach very high values for the reaction wheels. Due the constant straight uniaxial acceleration of spacecraft is the specific impulse of plasma thrusters higher comparing to combustion thrusters. The energy efficiency is significant higher also. Therefore, the consideration is focused on plasma thruster. The control of pulsed thrusters is described in [9]. This consideration deals primarily with the control structure and parallel simulation. Therefore, the design of the thruster as well as the geometry and dimensions are not a focus here.

Many kinds of thrusters exist today and much more are possible to realise. A focus with overview of list them all can be found in [10]. The gaseous ions enter the chamber at a predefined pressure and gets ionised there. The rough structure of engine includes propellant, power, electronics and control. The electrostatic field in the chamber generates plasma in the gas chamber, ionising it; causing by charged particles and ions get to the electromagnetic accelerator. The passed literature to the thrust measurement method is described in [11]. The use of the FPGA with the microcomputer has more advantages and robustness compared to the common variants. The parallel calculations are included [8] in the structure also. The base for the using advanced control structure is described in [17].

2.1. Rough Structure

The picture bellow shows a scheme of the thruster. In the electrostatic thruster first, a gaseous state and a nominal temperature must be achieved in the propellant chamber.

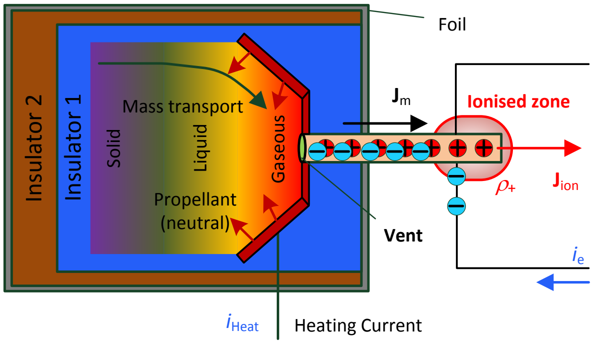

Figure 1. Ion thruster with the active controlled system.

A very small opening with the mass flow is controlled. The speed can be calculated by known temperature and pressure. The propellant current density represents a mass current density. The chamber itself is maximally insulated to achieve maximum efficiency. The mass flow gets into a zone with an electric field with the result; the electrons are detached with high ionisation rate. Since ions are much heavier, they fly directly into the accelerator. The electrons are led outside and thrown outwards. It should be noted that the first acceleration takes place in the propellant chamber. Depending on the temperature, this can already have a large share. When the opening is closed, the pressure increases isochronally with increasing temperature. According to Gay-Lussac, the following equation applies:

$$\frac{p}{T} = const. \qquad p = \rho_m \cdot R_s \cdot T \tag{1}$$

ρ_m … Mass density of matter (gas)

The heating system may be designed with several variants like heating coil, LED circuit, microwave or heating resistance. As an example, an application with iodine is presented in this work. The process in the chamber, the ionisation process and the acceleration process are generally to be distinguished. For the evaluation and design of the chamber, the usual theoretical consideration of gas has been made. Furthermore, the Navier-Stokes equations may be used also. After ionisation process is completed, an electrodynamic consideration will be used only. For sensing coated thermocouple up to 1200 ° C with silicone cable will be used. Temperature limit of the chamber is 1000 ° C. If the temperature gets a higher value, all heating processes must be stopped first. To distinguish are different kinds of densities. In the case of mass flow, J is marked with an "m" otherwise with "e" for electrons or ions.

3. PROPELLANT USE

The choice of propellant in an electric plasma propulsion system affects the performance of the engine. This includes thrust, the specific impulse, efficiency, power-to-thrust ratio. Generally, the complexity and cost of the entire system are decisive. For successful operation and sensible use of materials in space a propellant for ion thrusters should combine a low ionisation threshold with a high ionisation cross section to minimise the energy required to produce high density plasma. A high molecular weight may be required to reduce the amount of propellant in handling and storage qualities. Liquid or solid propellant can provide a higher density with a lower volume. High compatibility of system and low contamination potential for spacecraft and material is still required. Considering the fact that each propellant has some drawbacks, xenon has been used in few decades because it offers the best combinations of these properties. The main drawbacks are cost and density (compared to liquid and solid propellants).

Use of xenon (Xe) as propellant is the most widely for space applications, especially in Hall Effect (HET) engines. This is due to its special physical and chemical properties such as low energy of the first level of ionisation, high atomic mass and chemical inertness. However, this gas is extremely expensive due to its limited availability and its very expensive production process, and this aspect may be used in planning high velocities, such as cargo missions and orbital transfer missions are becoming a significant constraint.

Table 1. Several propellants with costs. ([1] and [2])

Propellant	**Ne**	**Ar**	**Xe**	**Kr**	**He**
Mass	20,20	39,90	131,3	83,80	4,00
State	Gas	Gas	Gas	Gas	Gas
Ionisation energies (eV)	21,56/ 40,96	15,76 27,63	12,13/ 20,97	14/ 24,36	24,59/ 54,41
Melting / Boiling point (K)	24,6/ 27,1	83,8/ 87,3	161,4/ 165,1	115,8/ 119,7	0,95/ 4,2
Critical Temperature (K)	44,57	150,7	289,7	209,5	5,2
Density (g/cm³)	$9{,}0\cdot10^{-4}$	$1{,}8\cdot10^{-3}$	$5{,}9\cdot10^{-3}$	$3{,}7\cdot10^{-3}$	$2{,}0\cdot10^{-4}$
Cost (per 100 g)	30 €	0,4 €	100 €	30 €	4,5 €

Use of Sodium (Na) has a relatively low ionisation level and thus can be ionised very efficiently. The problem with it is its aggressive chemical property and a high boiling temperature (883 °C). As possible propellant the Krypton (Kr) may be used also. However the substances that have a high molecular weight to save volume are proposed to be use first. In accordance with these criteria, analysis was accomplished for the following selected propellants also.

This includes Hydrogen (H2), Xenon (Xe), Krypton (Kr), Argon (Ar), Neon (Ne), Helium (He), Iodine (I), Mercury (Hg). Their properties are given for standard temperature and pressure, 273,15 K and 105 Pa. Therefore, the focus of this work laid on iodine. This is solid material and can therefore be integrated relatively easily. Iodine has following properties:

$\rho_{Im} = 4{,}93\ g/cm^3$ with $m_{amu} = 126{,}9\ u$

$T_{melt} = 386{,}9\ K \triangleq 113\ °C;$

$T_{boil} = 457{,}6\ K \triangleq 185\ °C$

First and second Ionisation energy levels have

$E_{i1} = 10{,}5\ eV \quad E_{i2} = 19{,}3\ eV$

Molecular mass of iodine has following equation

$$M_i = N_A \cdot m_i = 127\ \frac{g}{mol} \tag{2}$$

The specific gas constant of Iodine has

$$R_{is} = \frac{R_0}{M_i} = 0{,}0655\ \frac{J}{g\cdot K} \tag{3}$$

With following constants

$N_A = 6{,}022 \cdot 10^{23}\ \frac{1}{mol}$ Avogadro-constant

$R_0 = 8{,}314\ \ J/(mol \cdot K)$ Universell gas constant

The high pressure at the beginning, 258 MPa, results from the fact that the density cannot change initially. However, that changes a lot when the propellant leaves step by step more and more space in the chamber and thus the density will change. This allows the pressure to be controlled so that it does not exceed the maximum permissible. The solution is relatively simple; the chamber must have a small space in front of the opening to generate safely the gaseous density. Thereby the critical temperature of iodine is 819 K, critical pressure is 11,7 MPa.

$$\rho_{m,max}(800K) = \rho_{m0} \cdot \frac{p_{allowed}}{p_{imax}} = 0{,}38\ \frac{g}{cm^3} \tag{4}$$

At 800 K iodine in gaseous state, depending on the opening area, the speed of the particles flying out may be determined. Since the static pressure in the opening is equal to the dynamic pressure:

$$p_i = p_{idyn} = \rho_{Im} \cdot \frac{v^2}{2} \tag{5}$$

$$v_i(800\ K) = \sqrt{2 \cdot \frac{p_i}{\rho_{Im}}} = 323{,}5\ \frac{m}{s} \tag{6}$$

Since the temperature should be by 800 K in the chamber, the speed does not change by the density changes. One can calculate the mass flow density as follows, because the pressure outside is ≈ 0 Pa. There are two scenarios for the use:

1. (p at p_{max} by ca. 99 % of propellant)

$$\mathrm{J}_{im1} = \rho_{Imax} \cdot \mathbf{v}_i \tag{7}$$

$$|\mathrm{J}_{im1}| = 159{,}5\ \frac{kg}{s\cdot cm^2} \triangleq 159{,}5 \cdot 10^6\ \frac{kg}{s\cdot m^2}$$

2. (p at p_{min} by ca. 1 % of propellant)

$$\mathrm{J}_{im2} = \rho_{Imin} \cdot \mathbf{v}_i \tag{8}$$

$$|\mathrm{J}_{im2}| = 1{,}59\ \frac{kg}{s\cdot cm^2} \triangleq 1{,}59 \cdot 10^6\ \frac{kg}{s\cdot m^2}$$

This makes possible to set the maximum mass flow through the surface of the opening. However, this results in the ion density also, depending on the number of ionisations. One starts with a case where all the atoms fly out have been ionised:

1. (p at p_{max} by ca. 99 % of propellant)

$$|\mathrm{J}_{ie1}^{+}| = |\mathrm{J}_{im1}| \cdot \frac{e}{m_{atom}} = 1{,}21\ \frac{MA}{mm^2} \tag{9}$$

Result by first ionising energy and

$$|J_{ie1}^{++}| = |J_{im1}| \cdot \frac{e}{m_{atom}} = 2{,}42\ \frac{MA}{mm^2} \tag{10}$$

Result by second ionising energy.

2. (p at p_{min} by ca. 1 % of propellant)

$$|J_{ie2}^{+}| = |J_{im2}| \cdot \frac{e}{m_{atom}} = 0{,}12\ \frac{kA}{mm^2} \tag{11}$$

Result by first ionising energy

$$|J_{ie2}^{++}| = |J_{im2}| \cdot \frac{e}{m_{atom}} = 0{,}24\ \frac{kA}{mm^2} \tag{12}$$

Result by second ionising energy.

Design is going to the next step. The power of the device should not exceed 300 W at peak, but 100 watts at operating point. At the initially set voltage of 3 kV, the area of the opening, or use of the valve, should be evaluated first. The ion current has there a value by maximal power 33 mA. In the case 1 it would mean 1 µm width (too low). In case 2 it would mean 3 µm width (too low). The aim is to get width of round pipe with more than 100 µm by the same ion current of 33 mA. By the first and second ionising energy one gets:

$$|J_{ie}^{+}| = \frac{I}{A} = 3{,}3 \frac{A}{mm^2} \qquad |J_{ie}^{++}| = \frac{I}{A} = 6{,}6 \frac{A}{mm^2}$$

The value has a similar current density as in electrical installations. The mass flow has a value:

$$\left|J_{im}\right| \cdot \frac{m_{atom}}{e} = 4{,}34\ \frac{kg}{s \cdot m^2} \qquad \left|J_{im1}\right| = \rho_m \cdot v_i$$

To reach these values and not to exceed them there are two options given; reduce velocity or reduce density. To reduce velocity of iodine particles would mean that the impulse density will decrease also. The boiling temperature 457 K (183 °C), the temperature must be at least +100 K to ensure that iodine has become indeed gaseous. For velocity follows 966 km/h. The speed at a significant temperature drop goes down only minimally. In this step, the essential design decision can be made. It would be better to use a firmer material than to reduce the temperature. There is given a start speed and thus an impulse. Even more, it should be increased if possible, the speed, if possible over 1300 K.

$$\rho_m(33\,mA, 560K) = \frac{\left|J_{im0}\right|}{v_i} = 0{,}016\ \frac{kg}{m^3} = 16{,}2\ \frac{g}{m^3}$$
$$\rho_m(100\,mA, 560K) = 0{,}049\ \frac{kg}{m^3} = 48{,}5\ \frac{g}{m^3}$$

The lowest level of density (operation point) is about 25 times lower than atmospheric pressure. Thus, all key data are determined. The rest is the art of adjusting the heating so that the exit velocity is high enough as possible that iodine remains in the gaseous state. For the velocities a kinetic energy and the momentum per atom one can give as follows:

$$p = m_I \cdot v_{Thermal} = 6{,}81 \cdot 10^{-23}\ Ns \tag{13}$$

$$E_{kin} = \frac{1}{2} \cdot p \cdot v = 1{,}1 \cdot 10^{-20}\ Ws \tag{14}$$

As a rough comparison, the energy in eV one can give, depending on how often the iodine atom is ionised:

$$E_{kin(1)} = 68{,}8\ meV \quad E_{kin(2)} = 34{,}4\ meV$$

Thus, operating voltages would be necessary that are in the mV range only. The goal is to accelerate the iodine atoms to the energy of several kV. Here, the efficiency of ion thrusters is immediately clear that the propellant is used much more efficiently that the combustion, thermal use only.

In general, the exothermic reaction or endothermic reaction applies to burns. The rockets use exothermic reaction in which the energy is released and used for acceleration.

Here, the thermal (movement of the molecules) and radiation, which is mostly in the visible wavelength range. The chemical reaction rate is defined by the Arrhenius law:

$$k = A \cdot exp\left(-\frac{E_a}{R_0 \cdot T}\right) \tag{15}$$

Here, E_a is the energy of activation, R_0 is the universal gas constant, A experimental parameter.

4. DIFFUSION EFFECTS

The motivation to investigate the diffusion effects emerged during the first experiments. The results of them were strongly differing from the expected results. In principle, there were additional delays in force and current measurements to observe. Furthermore, the delayed signals of accelerator-, ionisation-currents had a fundamentally different temporal course and a higher system order than expected. In the plasma propulsion system there is a lot of conductive connections, elements and power transmitters, which occur due to the altering of the current values. They follow a distribution of eddy currents in the coils. The charge distribution in the ionisation chamber has diffusion behaviour also. In view of this there are to differ designed and natural systems with the nearly almost non-linear transfer functions.

The real-time calculation of diffusion effects would solve this problem. The successful design of such system includes any current and voltage forms. This may be integrated for control algorithm. This allows embedding these coupled systems in microcomputer structures. Another aim is the calculation of diffusion, which is caused by the induction and movement of plasma in the magnetic and electric field. In the magnetic guidance part or in other conducting regions several electrodynamic effects may occur simultaneously.

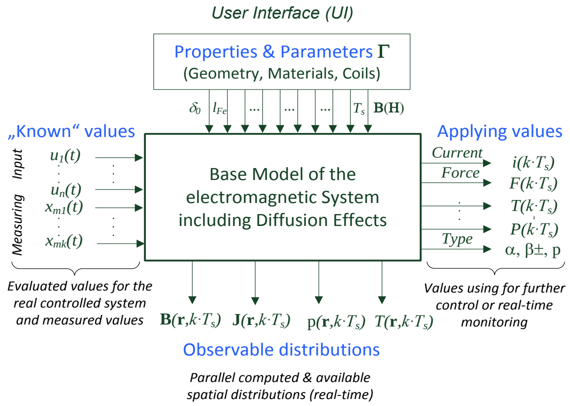

Figure 2. Real-time evaluation system with I/0 and distribution of fields.

These include electromagnetic diffusion (eddy current distribution) but electromagnetic waves (wave spreading) also which may cause a fault in neighbouring systems of spacecraft. Thus the diffusion has influence to the dynamics of the closed control loop and cannot be neglected. One-dimensional models are often a sufficient approximation to the exact distribution of eddy currents. The next picture below aim shows a realisation of parallel computing real-time system with the deposited model to evaluate and to monitor electrodynamic and thermal diffusion effects.

The proposed model includes Diffusion effects as well in a low frequency range (Heat Diffusion) as higher (> 10 kHz) frequency range. The output quantities are compared with the measured quantities and may re-adjust the uncertainties of the mathematical model. In the next step will the diffusion effects treated. There are distributions in real-time from the calculations available.

4.1. Diffusion by Fick's Interpretation

In the idealised case of nearly ideal thin dynamic charged particles the diffusion current density **J** can be evaluated according to the first Fick's law [13]:

$$\mathrm{J} = D \cdot \nabla\rho \tag{16}$$

D represents the Diffusion coefficient and ρ represents density. It can be a mass, concentration and charge density. Calculation of D with Einstein relation is described in [14]. The equation describes high dynamic diffusion and distribution of charge density and has no defined sources, like an electric potential with given charge density. This may be solved with given initial conditions, for ionisation effects, plasma distribution, pn-junction, biological and many other applied systems.

4.2. Extended Field strength

With all material constants for magnetic, electric field and using vector analysis one obtains the wave equation. From the perspective of systems theory a novel three-dimensional vector size **K** may be defined. Without making a mistake for further analysis follows:

$$\mathrm{K} = -\nabla\rho - \frac{1}{c^2} \cdot \frac{\partial \mathbf{J}}{\partial t} \tag{17}$$

This diffusion field K represents the proposed density drifting vector. A negative gradient of density appears in the equation. It is a reason to assume that the equation is directly equivalent to the extended first Fick's law:

$$\mathrm{J} = -D \cdot \mathrm{K} \tag{18}$$

By multiplying of charge density with c^2 follows an expression:

$$p_e = \rho \cdot c^2 \tag{19}$$

This is electrostatic pressure with c as a medium light speed. That may be defined as gas equation.

$$p_e = \rho \cdot R_s \cdot T \tag{20}$$

T is replacement for the temperature. This can be interpreted as vacuum electrostatic pressure with a temperature level. This process has similarity to the acoustic pressure.

The current density *J* is proportional to the drifting vector *K* and may be interpreted as Ohm's law. For coils material (copper) the diffusion coefficient *D* under normal temperature (300 K) equals $10{,}9 \times 10^3$ m^2/s. Compared to Particle Diffusion of Hydrogen ($D = 7{,}79 \times 10^{-5}$) the speed of diffusion ≈ 10^8 times is faster in a conductor.

4.3. Continuum Equations

I. Moving charge and current density [12]

$$\nabla \cdot \mathrm{J} + \frac{\partial \rho}{\partial t} = 0 \tag{21}$$

This equation is well known; charge conservation is similar to mass conservation and continuum equation has the same notation. Now one has to get a look to

II. Lorenz-gauge

$$\nabla\cdot \mathrm{A}+\frac{\partial\phi}{\partial t}=0 \quad \text{with} \quad \phi=\frac{1}{c^2}\cdot\varphi \tag{22}$$

The electrical potential can also be interpreted as a first approximation as pressure, which only exists in vacuum. The equation becomes a continuum equation with the associated potential density ϕ. Now, the following relationships arise, flux (transport) of potential as the electric current represents the charge transport. From equation II follows directly:

$$\oiint \mathrm{A}\cdot d\mathrm{O}+\frac{\partial}{\partial t}\left(\iiint_{(V)}\phi\cdot d\tau\right)=0 \tag{23}$$

One can easily see that it can be used to define a current that represents the potential flow.

$$F=\iint_{(O)}\mathrm{A}\cdot d\mathrm{O} \qquad S=\iiint_{(V)}\phi\cdot d\tau \tag{24}$$

That means that the charge S of the electric potential must exist. It follows that the volume integral of the electric potential must be finite. Then the same applies to the A - vector and the potential field:

$$\Phi=\oint \mathrm{A}\cdot ds \qquad I=\iint_{(O)}\mathrm{J}\cdot d\mathrm{O} \tag{25}$$

Now one compares the equation for the potential charge and immediately recognises that the equation already exists there. In the simplest and assumed case follows is the current density directly proportional to the vector potential. In which case or medium this may be exist is not easy to analyse, but a connection between the J and A field may be seen, both from continuum equations and field equations. It is a part of the further research.

4.4. Relativistic Diffusion Equations

First, the potential equation can be specified. It describes the propagation of the potential in a conducting medium, but does not describe the ionisation process directly. It follows:

$$\Delta\varphi=a^2\cdot\frac{\partial\varphi}{\partial t}+\frac{1}{c^2}\cdot\frac{\partial^2\varphi}{\partial t^2} \tag{26}$$

$\kappa\cdot\mu=a^2$ Eddy current coefficient.

Although this equation is used as a telegraph equation [12], it is an evanescent process; this process is diffusion. In simplified case for eddy currents of the conducting material the wave propagation with the time delay r / c can be neglected:

$$\Delta \mathrm{A}=\kappa\cdot\mu\cdot\frac{\partial\mathbf{A}}{\partial t} \qquad \Delta\varphi=\kappa\cdot\mu\cdot\frac{\partial\varphi}{\partial t} \tag{27}$$

And the similar to potential the relativistic diffusion equation exists also

$$\Delta\rho=b^2\cdot\frac{\partial\rho}{\partial t}+\frac{1}{c^2}\cdot\frac{\partial^2\rho}{\partial t^2} \tag{28}$$

$\frac{1}{D}=b^2$ Diffusion coefficient.

This derivation of the relativistic for the charge density distribution was shown in [19] and has a wave character also. In simplified case the time delay r / c can be neglected also:

$$\Delta\rho=\frac{1}{D}\cdot\frac{\partial\rho}{\partial t} \tag{29}$$

Since the potential equation describes the distribution of the potential and thus also the associated currents and eddy currents, the relativistic diffusion equation for the charge gives the opportunity to describe the ionisation and radiation processes also. These can arise in many ways and above all through the radiation.

The heat diffusion process has a similar reaction according to Plasma and Eddy currents; the necessary equations can be looked up in [16]. One of them is:

$$\Delta T=d^2\cdot\frac{\partial T}{\partial t}+\frac{1}{c^2}\cdot\frac{\partial^2 T}{\partial t^2} \tag{30}$$

d^2 Heat Diffusion coefficient.

The equation is a relativistic Heat equation. Since the time constants are relatively large, the thermal control is not such a difficulty to realise. The equation is particularly useful when the thermodynamic processes are very fast; in the case propagation velocity is noticeable in comparison to the speed of light.

In the field of avionics there is an amount of systems depending not only on electrodynamic effects but rather on the thermal effects. Thus, for example the PDU (Power Distribution Unit) produces waste heat and warms itself and the direct environment. Real-time realisations have to be optimised. Special mathematic algorithms may be used. One of them is developed by Schönhage [15].

5. ANALYTICAL APPROACH

5.1. Common charge of Propellant

The amount of charge is crucial for the design and later use of spacecraft. For this, the first Maxwell equation may be considered:

$$Q_0 = \oiint \mathrm{D}_0 \cdot dO = \iiint \rho_0 \cdot d\tau \qquad (31)$$

$\rho_0 = n_0 \cdot e$
n_0 Ionised zone in $1/\mathrm{m}^3$

For the material law may be used the second Maxwell equation. It is important to distinguish whether the ionised zone is positive or negative charged. Usually, the electrons are separated from the atoms by an external force and leaving behind the positive charged ions. At normal state, the charge level is neutral:

$$Q_0 = \iiint (\rho_+ + \rho_-) \cdot d\tau = 0 \qquad (32)$$

To calculate the electric current from the ions or electrons, several steps must be done.

5.2. Escape from the zone (Ionising zone)

By leaving of the ionisation zone, the continuum equation applies to charge will be used. For a detailed description, states such as solid, liquid and gaseous are necessary. For this purpose temperatures have to be calculated or indicated. Likewise, pressure and mobility of the electrons are important. As the process progresses, the forces between the atoms are important. During ionisation, positive charged atoms remain in the chamber.

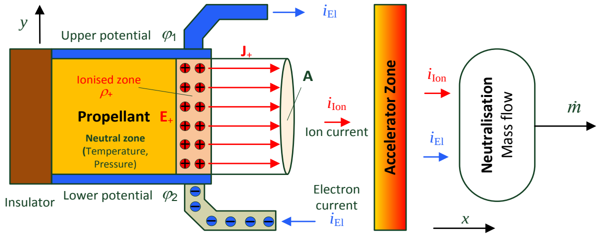

Figure 3. Generation and guidance of ions and electrons.

They interact with each other and generate repulsive forces. When the forces are greater than the chemical, they separate and move towards the open outlet. These processes have to be controlled. There are two mechanisms for ion current flow by in the propellant chamber possible, the drift current (Ohm's Law) and the diffusion current (Fick's Law). It should be noted that in the case of a solid propellant the ion current is possible in a high dynamic and adapted process only. One may consider only the ionised and a fixed predetermined applied potential (upper and lower) should be considered first. If the ionisation zone is considered to be infinitesimally thin, a Poisson differential equation can be given between the potentials approximately.

The solution for the potential and the E-field is by given height *h* of propellant and describes a generated field that pushes out of the ions up and down. The potential on the branches will be not generated additionally. In the case of applied potential on branch 1 and 2 with shifted boundaries and with Phi$_1$ = 0 follows:

$$\varphi(y) = -\frac{\rho_+}{\varepsilon_0 \cdot \varepsilon_r} \cdot \frac{y^2}{2} + \left(\frac{\rho_+}{\varepsilon_0 \cdot \varepsilon_r} \frac{h}{2} + \frac{\varphi_2}{h} \right) \cdot y \qquad (33)$$

And accordingly for the electric field follows:

$$E(y) = \frac{\rho_+}{\varepsilon_0 \cdot \varepsilon_r} \cdot y - \frac{\rho_+}{\varepsilon_0 \cdot \varepsilon_r} \frac{h}{2} + \frac{\varphi_2}{h} \qquad (34)$$

The formula describes an electric field shifted by the applied potential-field. The electron flow is the same as the neutralisation flow. This is one branch of charge flow. In case the electrons were successfully (high rated) separated with ionisation energy and shot out to space, stay charged ions that repel. The output electron beam is the current to neutralise the ion current. The electric current of electrons has to be the same as the ion current. The ion flow is the other branch. To build the calculation models, it is first necessary to understand that a generated electric current (ions) will not change through acceleration. The current density is the same. Interesting is the fact, that charge density will decrease and velocity of particle flow will increase. The transmitted power in the acceleration zone is the current multiplied by the voltage:

$$P_{Ion} = U_{Acc} \cdot i_{Ion} \qquad (35)$$

Thus, it is obvious that only the generated current and the voltage (potential difference) are important for the power output. In most applications for the plasma propulsions, the electrons are not used to contribute the thrust. That can be shown in a different way also. In the simplest case, the acceleration zone is round with a length *l* along the x-axis. The current in this zone remains constant because it flows through the concept of uniformity only at the output:

$$\frac{\partial \mathrm{J}_{x0}}{\partial x} + \frac{\partial \rho_0}{\partial t} = 0 \qquad (36)$$

One can recognize in the equation that only in the case of an ion collection in the accelerator zone the current at the output would not grow, as reservoir for example. Since the accelerator zone shoots out the ions, there is none in it. In the stationary state, the charge distribution remains constant over time. Thus, the current density remains the same, although the particle flux is accelerated. The charge distribution may be of interest, however, within the accelerator zone. This question can only be explained with Newton's laws of physics. To this may be considered that the E-field is constant and the charge density of ions is initially low to repel, so that it can be neglected. First, one calculates the acceleration vector for one electron (simple case):

$$\mathrm{a}_e = \frac{e}{m_e} \cdot \mathrm{E} \tag{37}$$

m_e … Mass of electron

m_{Ion} … Mass of ion

The ratio of elementary charge to electron mass is constant. The recommended value is e/m_e = −1,75882×10^{11} C/kg. The ratio can be considered statistically, for a quasi-continuous electron current. In this case, instead of the elementary charge, the charge density and instead of the mass, the mass density may be used. Thus, it becomes apparent that regardless of value of the charge density in the electron current, the acceleration by given E-field in the zone remains the same. Just one looks at the electric power density in the accelerator zone. The chosen power formula is for radiation of electric power:

$$P = \iiint p \cdot d\tau = \iiint \mathrm{J} \cdot \mathrm{E} \cdot d\tau \tag{38}$$

$$p = \mathrm{J} \cdot \mathrm{E} \text{ Power density in W/m}^3$$

With constant field sizes and simple volume design follows:

$$P = \underbrace{E_x \cdot l}_{U_{Acc}} \cdot \underbrace{J_x \cdot A}_{I_{Ion}} = U_{Acc} \cdot I_{Ion} \tag{39}$$

The power depends on the current of the ions and the accelerator voltage only. The complete power is radiated, in the form of ion and electron radiation. Now the speed vector of electrons and ions should be calculated.

$$\mathrm{v}_e = \int \mathrm{a}_e \cdot dt = \int \frac{e}{m_e} \cdot \mathrm{E} \cdot dt = \frac{e}{m_e} \cdot \mathrm{E} \cdot \Delta t \tag{40}$$

The time can be calculated and the displacement vector of the electrons can be used.

$$v_{xe} = \sqrt{\frac{e}{m_e} \cdot 2 \cdot \underbrace{\Delta x \cdot E_x}_{U_{Acc}}} = \sqrt{\frac{e}{m_e} \cdot 2 \cdot U_{Acc}} \tag{41}$$

The speed can also be calculated from the kinetic energy and will come to the same result. The speed distribution can be calculated with it. The charge distribution can be calculated from the current density:

$$J_{xe} = \rho_{Ion}(x) \cdot v_{xe}(x) = const. \tag{42}$$

It follows the same procedure applies to the ions:

$$\rho_{Ion}(x) = \frac{J_{xIon}}{C_{Ion}} \cdot \frac{\alpha}{\sqrt{x}} \qquad x \in [0, l] \tag{43}$$

Since the magnetic energy

$$W_m = \frac{1}{2} \iiint \mathrm{J} \cdot \mathrm{A} \cdot d\tau \tag{44}$$

Compared to the electrostatic neglected, the speeds related to light speeds can be neglected. For this can be used following formulas for the energy:

$$W_e = \frac{1}{2} \iiint \rho \cdot \varphi \cdot d\tau \qquad \text{since} \quad W_m \ll W_e \tag{45}$$

The electrons or ions also create a voltage in the accelerator zone, which can also be neglected. Thus, the applied voltage can be easily specified from the Laplace equation.

$$\varphi(x) = E_x \cdot x = U_{Acc} \cdot \frac{x}{l} \tag{46}$$

The accelerator has a length of 1 cm and a voltage, for example, 2 kV. It should generate a power of 100 watts. The necessary current is 50 mA. The cross-sectional area of the ion current is 1 mm². Thus, the current density is in the accelerator:

$$J_{xIon} = 6{,}6\ \mathrm{A/mm^2}; \qquad E_{x0} = 200\ \mathrm{kV}/m;$$
$$E_{x0} = E_{xIon}; \qquad J_{x0} = n_0 \cdot e \cdot v_0.$$
$$\rho_{Ion}(x) = \frac{6{,}6}{C_{Ion}} \cdot \frac{1}{\sqrt{x}} \qquad x \in [0, l] \tag{47}$$

Now that the charge distribution is known, the thrust and the momentum can be calculated. The nominal thrust for 100 watts is for iodine 3,63 mN. The chamber for the propellant has a weight 1,5 kg. Helium would reach 0,64 mN only, but with xenon it is possible to reach 3,7 mN.

It is minimally bigger than iodine only. Since the expense of manufacturing and controlling the propellant out of the chamber is relatively high, xenon is not the best candidate, especially if the high price is considered. Xenon is about 20 times more expensive than iodine.

6. RADIATION EFFECTS

With an additional observer system for the radiation, the disturbances as ionisation or x-rays may be determined. This provides the estimation of the more precise states (vectors) for the control and additionally the estimation of the disturbances that arise due to radiation effects.

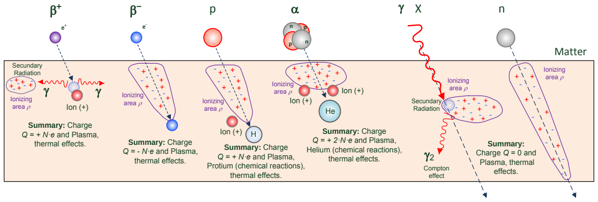

Figure 4. Types of ionising and non-ionising radiation.

The results have also shown that an observer system may be developed specifically for the real-time calculation (estimation) of the radiation effects only. Designed electronic systems may take damage caused by impacts with charged particle flux in Space. TID (Total Ionising Dose) of 1 Gy and Single Effect Transient (SET) free operation up to 50 MeVcm²/mg may assure certain functions. Single-Event Latch-up (SEL) results on the placement of several transistors in the shared substrate of an integrated circuit; ionising radiation can activate an additional parasitic thyristor. This short circuit between transistors can destroy the device if no protection measures in place. Single-Event Burnout (SEB) on the other hand increases current between drain and source of a MOSFET and destroys the component in a short time. A Single-Event Gate Rupture (SEGR) can destroy a dielectric of semiconductor also.

In order to be able to react to these processes, it must be calculated within a shorter time that ionising radiation is present. For this purpose, sensors can be used for the realistic evaluation of the diffusion and ionising effects of the test system. For this purpose, the Peltier element is used for the evaluation of the dynamic temperature increases (dT/dt), from which a measure of the ionisation processes and thus radiation will be detected. In addition, the piezo element can be used to record highly dynamic vibrations and oscillations to absorb impacts of charged particle flux.

All available sensors can be used to calibrate the spatial distributions also. By measured value of size and known location of the sensors, the entire distribution in space can be calculated retroactively or more accurately. With the formation the type of ionisation and the direct effect to the systems and thus possible prevent processes can be activated up to the shutdown. The results show possibilities to perform more qualitative and faster simulations with calculations independent of space-systems and radiation environment also.

The results show possibilities to perform more qualitative and faster simulations with calculations independent of space-systems and radiation environment also. If the additional charge is bigger than a device specific critical charge, also collected by source or drain it's possible to influence the dynamic behaviour of the device. For the electromagnetic propulsion engine it is a very useful extension to monitor the states by parallel calculation to use a real-time discrete simulation. Too complicated relations may usually be simplified.

7. RESULTS & DISCUSSION

For the electromagnetic propulsion system, various mathematical models have been created. The particular difficulty was in the consideration of the three-dimensional field generation, as well as in the decoupling the electromagnetic and spatial quantities. The quality of computation is high enough to be able to reach the aim, if FEM software is not real-time capable or completely missing. This applies multiple calculations, parameter studies, parameter optimisation and especially simplifications. It has been shown that iodine is very well suited for use as a propellant. It has several advantages, especially when it comes to boiling temperature.

A consideration of the electrodynamic effects for arbitrary signals at high sampling frequencies could be achieved. For verification, an additional FEM model has been created. The analytical investigations on electrodynamic effects for the field guidance in the ionisation chamber and the outer coils have shown that the accuracy of the results is sufficient to determine the influence of the diffusion. Furthermore, it could be shown, how the modelling of the diffusion may be simplified. It has also been found that the diffusion has positive effects on the stability and controllability of the controlled system, because the phase margin for the entire frequency range is kept to a maximum.

After reaching the cut-off frequency, the phase shift remains at -45 °. In addition, the field ripple is additionally smoothed with converter-power supplies.

8. CONCLUSION

First of all, it can be mentioned that the propellant will be chosen in the solid aggregate state in the near future. This makes it possible to realise more compact and safe engines.

The simulation environment in space may evolve in next years. On the one hand for own needs during flight, on the other hand for the support of the research.

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10. ABBREVIATIONS and ACRONYMS

LTI	Linear time invariant
PDE	Partial differential equations
DMO	Digital mathematic operations
LEO	Low Earth Orbit
FPGA	Field Programmable Gate Array
I/O	Input / Output
PWM	Pulse Width Modulation
μ	Magnetic Permeability of material
ε	Electric Permittivity of material
κ	Conductivity
μ_e	Mobility of charged particles (Plasma)
k_B	Boltzmann's constant
δ_0	Air gap
K	Density drift vector (vector)

B	Magnetic flux density (vector)
J	Current density (vector)
E	Electric field (vector)
A	Vector potential (vector)
ρ	Charge density
ρ_m	Mass density
φ	Electric potential
J_{Diff}	Diffusion Current density (vector)
J_{Drift}	Drift Current density, caused by E-field
D	Eletric displacment field (vector)
F	Mechanic force (vector)
Δ	Laplacian Operator
$u(t)$	Voltage (time depending)
$i(t)$	Electric Current (time depending)
T_1	Time constant of delay, first order
T_t	Operating time constant
T	Temperature in K
TID	Total Ionising Dose
SEL	Single-Event Latch-up
SEB	Single-Event Burnout
SEGR	Single-Event Gate Rupture

11. VITAE

Alexander NORBACH is highly interested in the field of plasma generation and of the accelerator design for various applications and has many years of experience in the computation and investigation of electromagnetic fields. He performed the most advanced electrodynamic calculations, involving nonlinear effects.

In 2019 brings along a successfully completed PhD with the focus on electric propulsion, control and power electronics. He was a technical project manager in the development and industrial technology transfer of a novel three-phased linear transverse flux machine at the Institute for Electric Drives, Power Electronics and Devices (IALB). During 6 years he worked as a research assistant at the IALB and got more involved in EM field calculations, systems theory, simulations, embedded systems, programming, electric propulsion technology, drive and circuit design, field-oriented and adaptive control.