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Research Article

Evaluation of Two-vs-One Air Combats Using Hybrid Maneuver-Based Framework and Security Strategy Approach

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Abstract

In this paper, we aim to develop a simulation platform and maneuver selection methodology for two-vs-one air combats. It consists of the aircraft performance model, aerial combat maneuvers and game theoretical decision strategy. The performance parameters of F-16 are used to implement the aircraft dynamics. The hybrid system theory underlies the modeling methodology of the aerial combat maneuvers, which are the possible options of the combatants. The decision strategy is derived from security policy approach in game theory using some metrics that present the air superiority. The definitions of these metrics and evaluation strategy are presented. The verification of the models and developed decision processes are given through the simulations.

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Hibrit Manevra Tabanlı Yapı ve Güvenlik Stratejisi Yaklaşımı ile 2x1 Hava Muharebelerinin Değerlendirilmesi

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Öz

Bu makalede, 2'ye karşı 1 hava muharebelerinin simülasyonunun yapılabileceği bir platform tasarlanmıştır. Tasarlanan platform uçak performans modeli, hava muharebe manevraları ve oyun teorisi kapsamlı karar stratejisini içermektedir. Uçak performans modeli F-16 uçağının performans parametreleri ile kullanılmıştır. Muharebe manevraları hibrit sistem modellemesi yardımıyla oluşturulmuştur. Bu manevralar hava muharebesinde pilotların olası seçimlerini ifade etmektedir. Bu çalışmada, hangi manevranın uygulanması gerektiği hava üstünlüğünü ifade etmek için kullanılan bazı metriklerin oyun teorisi yaklaşımları ile değerlendirilmesi ile elde edilmektedir. Değerlendirme stratejisi ve 2'ye karşı 1 hava muharebeleri için hava üstünlüğünü ifade eden metriklerin tanımlanması bu çalışmanın kapsamı içerisindedir. Çalışmanın son aşamasında modeller ve karar süreci simülasyonlarla tahkik edilmiştir.

1. INTRODUCTION

The air superiority refers to constituting sovereignty on airspace over enemy's air force. When a combat team outperforms the enemy's force and gets the air superiority, it takes a chance to shoot down the opponent. Therefore, air combatants derive various strategies to get air superiority that could bring victory. However, during combat, the enemy's force also tries some strategies to take advantage. The fighter team should predict the enemy's maneuvers and define their maneuvers according to the opponent's possible strategies. Thus, it is an active process that contains the prediction of challenger's possible strategies and reevaluation of choices. In this process, the main question is how can the combat team evaluate the opponent's actions and update their strategies? Air combat modeling is about this issue. It contains the evaluation of the enemy's movements and generation of actions to get air superiority. When the modeling environment is constructed, it can be used for decision support to fighter pilots, autonomous air combat and analyzing combat scenarios.

There have been several studies to model the aerial combat. The well-known approach is the presentation of the problem as a pursuit-evasion game. In this kind of approaches as in [9], a partial differential equation derived from the Hamilton-Jacobi equation is solved to determine the reachable sets that present the solution space. However, in this kind of approaches, the computation time is the real problem. The rule-based air combat simulation is an alternative approach such as represented in the report [5]. A rule-based logic is designed in this report. Besides the rule-based modeling methodology, the influence diagram is also used as a modeling approach. The authors of the study [15] design an influence diagram to simulate and analyze pilot decision process in one-on-one aerial combat. Moreover, the game strategic approaches are also demanded to model the decision process. Using elemental maneuvers such as steady flight, max load factor turn etc., Austin et al. [1],[2] design a game theoretic methodology that has a deterministic strategy to model and simulate the air combat between helicopters. In master thesis of McGrew [8], he also uses elemental maneuvers such as roll left, roll right and maintain the current roll. Using an approximate dynamic programming scheme, an algorithm is designed to support the aerial combat at fixed flight level. However, in these studies, the real aerial combat maneuvers are never used to determine the choices of the combatants. They generally use elemental maneuvers such as turn, go along etc. or a subset of these elemental maneuvers as input space. And, none of them is capable of simulating two-vs-one air combats.

In this work, we aim to develop a simulation platform for air combat formations, which we have focused especially on two-vs-one air combats. This platform allows us to run air combat simulations through searching decision state space of aircraft for the given metric for air superiority. In the platform, as a generic model, we have utilized F-16s and hybrid system theory to model decision logic of the aircraft. The decision strategy is obtained through game theory for the given air superiority metric. For the validation purpose, we have shown the simulation results of the maneuver generation methodology in the developed platform.

The rest of the paper is organized as follows. The aircraft model is described in section 2. Then, aerial combat maneuvers are presented as hybrid automata in section 3. Section 4 contains the quantification methodology of the combatant's objectives. In section 5, the security strategy is presented to evaluate the game. The implementations and simulation results are given in section 6. Then, section 7 consists of concluding remarks.

2. AIRCRAFT MODEL

The dynamics of aircraft is described in the state-space form by the following set of equations:

$$\dot{x}_1 = V\cos(\psi)\cos(\gamma) \tag{1}$$

$$\dot{x}_2 = V\sin(\psi)\cos(\gamma) \tag{2}$$

$$\dot{h} = V\sin(\gamma) \tag{3}$$

$$\dot{V} = -\frac{C_D S\rho V^2}{2m} - g\sin(\gamma) + \frac{T\cos(\alpha)}{m} \tag{4}$$

$$\dot{\psi} = \left(\frac{C_L S\rho V}{2m} + \frac{T\sin(\alpha)}{mV}\right)\frac{\sin(\mu)}{\cos(\gamma)} \tag{5}$$

where, the model has three control inputs ($[\delta, \mu, \gamma]$) and five state variables ($[x_1, x_2, h, V, \psi]$). In this system, throttle level (δ) is used to adjust the thrust (T). There is an engine model to generate the thrust (T) as a function of throttle level (δ), Mach number and altitude for F-16 in NASA report [10] and Steven's book [14]. This engine model and other performance parameters of F-16 are used in this study to simulate the aircraft motion. Whereas there is a first-order lag between commanded engine power level and power level in original model, we do not use this lag in this study. Using three look-up tables in [10],[14], the thrust is calculated according to Mach number, altitude and power level that is driven by throttle level without a time lag. Besides engine model, there is a proportional controller to hold the speed at desired value. This controller calculates the necessary throttle input, which is symbolized as:

$$\delta = f_{spd}(V_{desired}) \tag{6}$$

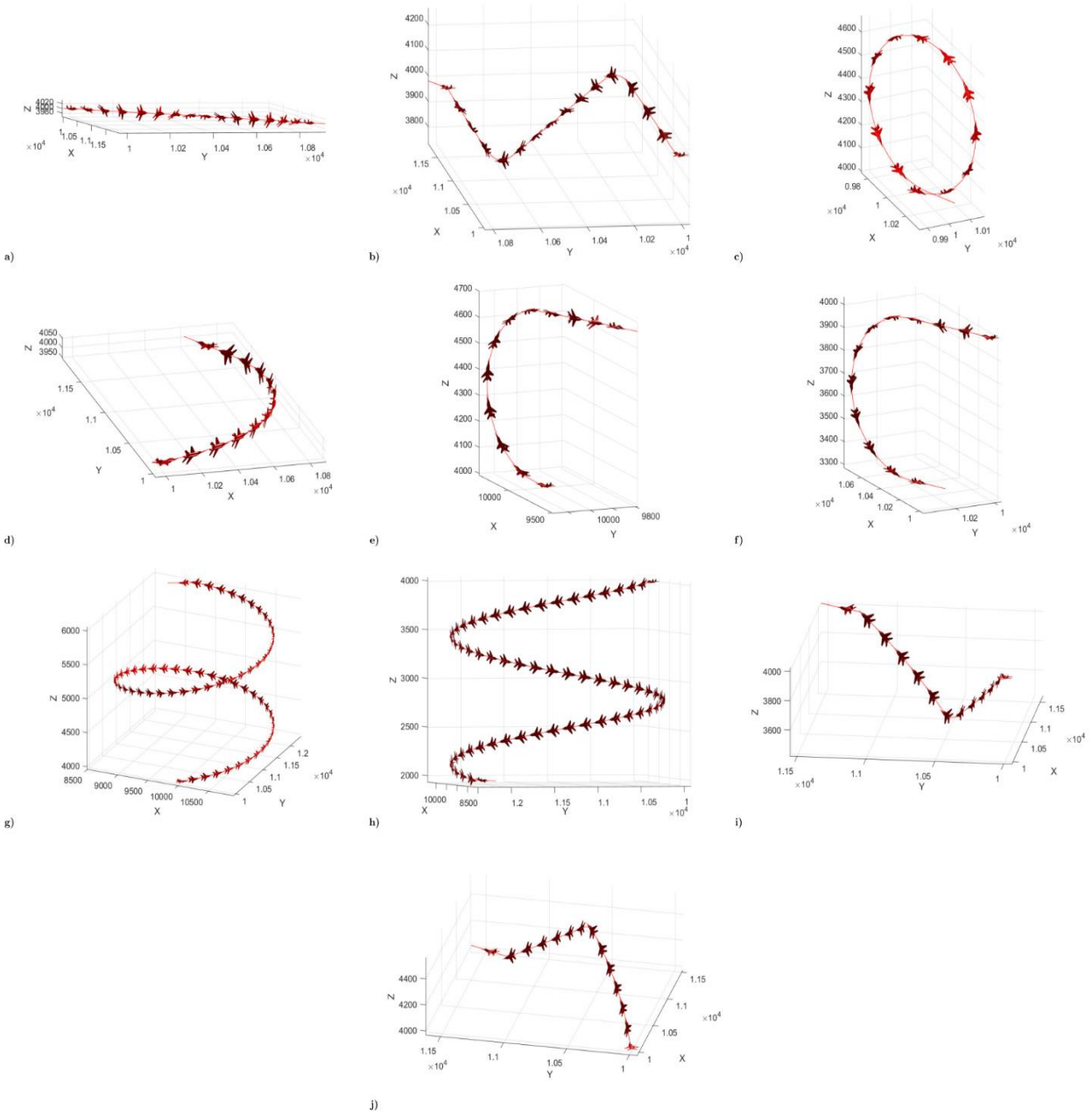

Figure 1. Aerial Combat Maneuvers in Maneuver Library: a) Aileron Roll, b) Barrel Roll, c) Loop, d) Break Turn, e) Immelmann, f) Split S, g) Vertical Spiral, h) Spiral Dive, i) Low Yo-Yo, j) High Yo-Yo.

In this study, C_L is determined as follows:

$$C_L = \frac{2mg\cos(\gamma)}{\rho V^2 S\cos(\mu)} \tag{7}$$

C_D and α are described as functions of the C_L as shown in (8) and (9). These functions are formulated using aerodynamic data in NASA report [10].

$$C_D = f_{C_D}(C_L) \tag{8}$$

$$\alpha = f_\alpha(C_L) \tag{9}$$

At the rest of the paper, this aircraft dynamic model is denoted as $\dot{x} = f(x,u)$, where x is the state variables and u is the control inputs.

3. AERIAL COMBAT MANEUVERS

A set of aerial combat maneuvers is used to specify the input space during aerial combat. These maneuvers, which are illustrated in Fig. 1, are the possible choices of the fighters during combat. The hybrid system theory is used to generate combat maneuvers. Each of them is modeled as a hybrid automaton. The hybrid automaton can be specified by a graph whose edges represent discrete transitions and vertices represent continuous activities, which are modeled by differential equations. The syntax [7], [11] of hybrid automata are defined as follows.

A hybrid automaton H is a collection $H = (Q, E, X, Init, Flow, Inv, G, R)$, where

- Q is a set $\{q_1, q_2, \ldots, q_n\}$ of discrete variables (states) that represent control modes of the hybrid system
- $E \subset Q \times Q$ is a collection of discrete transitions that represent discrete changes of control mode in the hybrid system
- X is a set of continuous variables
- $Init \subseteq Q \times X$ is a set of initial states
- $Flow: Q \times X \rightarrow TX$ is a vector field
- $Inv: Q \rightarrow P(X) := 2^x$ assigns to $q \in Q$ an invariant set
- $G: E \rightarrow P(X)$ assigns to each $e = (q, q') \in E$ a guard
- $R: E \times X \rightarrow P(X)$ assigns to each $e = (q, q') \in E \text{ and } x \in X$ a reset relation

where $\boldsymbol{P(X)}$ indicates the power set or set of all subsets of $\boldsymbol{X}$. Hybrid automata define possible evaluations of their state. Roughly speaking, an automaton starts from an initial value $(\boldsymbol{q_0}, \boldsymbol{x_0}) \in \boldsymbol{Init}$ and continuous variable $\boldsymbol{x}$ is updated according to differential equation

$$\dot{x} = f(q_0, x) \tag{10}$$

$$x(0) = x_0 \tag{11}$$

the discrete state q remains constant as $q(t) = q_0$ until continuous variable x reaches the quard $G(q_0, q_1) \subseteq \mathbb{R}^n$ of some edge $(q_0, q_1) \in E$. Then, the discrete state changes value to q_1 and the continuous variable may get reset to some value in $R(q_0, q_1, x) \subseteq \mathbb{R}^n$. After this discrete transition, continuous evaluation resumes and the whole process is repeated.

Aerial combat maneuvering is the combination of several simple maneuvers [12], [13]. To simulate the more complicated maneuvers, a maneuver library that consists of 10 different combat maneuvers will be used in this study [4]. The hybrid system descriptions of the maneuvers are presented at the rest of the section.

3.1. Aileron Roll

An Aileron Roll refers to a full $360°$ revolution about the longitudinal axis while maintaining the flight level and flight vector. Hybrid automaton of Aileron Roll is presented in Fig. 2.

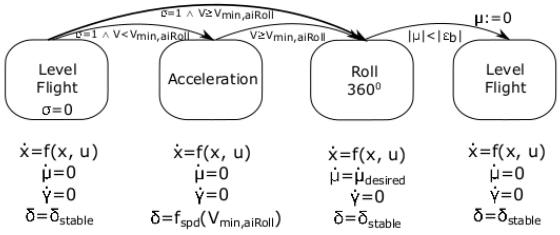

Figure 2. Hybrid Dynamics of Aileron Roll Maneuver.

In the hybrid automaton, $Q = \{LevelFlight, Acceleration, Roll360°, LevelFlight\}$ and the set of differential equations for each discrete state is illustrated at the below of related discrete state in Fig. 2. Transitions, guards and reset relations are also portrayed and reset relations are symbolized as $:=$. The initiation of the maneuver is controlled by a discrete input variable σ. The other input parameters are $\dot{\mu}_{desired}$ and δ_{stable}. ϵ_b corresponds to error tolerance for the bank angle and it is defined as $\dot{\mu}_{desired}\Delta t$. δ_{stable} denotes the throttle value that is necessary to keep the aircraft's speed at a desired value.

Although the discrete states use the same aircraft dynamics, they have different control mechanism to generate the maneuver. In the $LevelFlight$, the aircraft follows a straight-path at a fixed altitude because of setting the bank and path angle rates to zero as symbolized with $\dot{\mu} = 0$ and $\dot{\gamma} = 0$. The only difference between $LevelFlight$ and $Acceleration$ is speed controller. Whereas there is a speed controller in $Acceleration$ state for setting the speed to a specific value by adjusting the throttle, the throttle is set to a defined value without a speed controller in the $LevelFlight$ state. The $Roll360°$ state corresponds to setting the bank rate to a desired value for $360°$ revolution about the longitudinal axis.

3.2. Barrel Roll

In a Barrel Roll, the aircraft rotates both in its longitudinal and lateral axes, while in case of Aileron Roll, the rotation is only about the longitudinal axis. Because of the rotation, when compared with level flight without roll, more drag is generated; consequently, the Barrel Roll is used in many maneuvers to reduce the aircraft's speed.

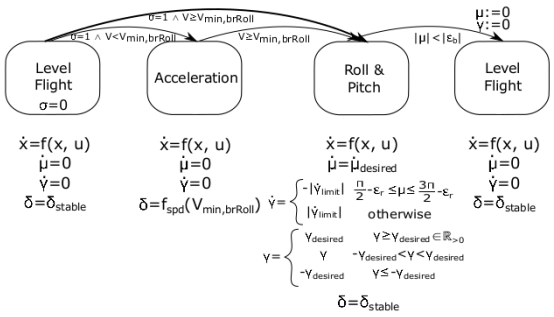

Figure 3. Hybrid Dynamics of Barrel Roll Maneuver.

In the hybrid automaton of the Barrel Roll, $Q = \{LevelFlight, Acceleration, Roll\&Pitch, LevelFlight\}$, the differential equations and other parameters are presented in Fig. 3. The initiation of the maneuver is controlled by a discrete input variable σ. The other input parameters are $\dot{\mu}_{desired}$, $\dot{\gamma}_{limit}$ and δ_{stable}. $\gamma_{desired}$ is standardized as $\pi/6$ for the Barrel Roll. $\dot{\gamma}_{limit}$ corresponds to maximum or minimum value of the allowed rate of path angle. It can be positive or negative. It can be taken according to aircraft performance limitations or can be defined by user that must be smaller than the aircraft performance limits.

The main difference between the Barrel Roll and Aileron Roll automaton is the third discrete state. The Barrel Roll uses $Roll\&Pitch$ as the third state, whereas it is $Roll360°$ in the Aileron Roll. The $Roll\&Pitch$ state involves the $Roll360°$ state that corresponds to setting the bank rate to a desired value for 360° revolution about the longitudinal axis. In addition to the rotation on the longitudinal axis, the $Roll\&Pitch$ state contains the rotation on the lateral axis by changing the flight path rate according to current bank angle of the aircraft. A barrel roll is illustrated in Figure 1.b.

3.3. Loop

In a Loop maneuver, the aircraft follows a circular path in the vertical plane, while the lateral axis of the aircraft remains horizontal. The hybrid automaton of the Loop maneuver is illustrated in Fig. 4.

In the hybrid automaton of the Loop, input variable σ is used to initialize the maneuver. The other input parameters are $\dot{\gamma}_{desired}$ and δ_{stable}. ϵ_p is the error tolerance for the flight path angle and it is defined as $\dot{\gamma}_{desired}\Delta t$. Additionally, this hybrid automaton has reset relations at the transitions from $Loop1^{st}Guarter$ to $Loop2^{nd}Guarter$ and from $Loop3^{rd}Guarter$ to $Loop4^{th}Guarter$. In the first one, the heading angle is decreased by π and the bank angle is set to π; while, the heading angle is increased by π and the bank angle is set to 0 for the second reset relation. These reset relations simulate the instantaneous heading change because of the rotation on the vertical plane.

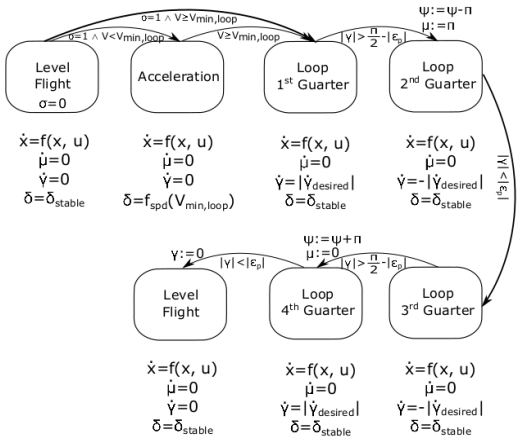

Figure 4. Hybrid Dynamics of Loop Maneuver.

The Loop automaton contains 4 consecutive Loop states that are implemented by setting the path angle rate. The sign of the path rate is defined according to sequence of the quarter. The transitions between Loop states are described according to the path angle. In the first quarter, the aircraft climbs with a constant path angle rate until the path angle reaches to $\pi/2$. Then, the second quarter, which has negative path angle rate, is begun that causes to decrease in the path angle, while the aircraft is still climbing. When the path angle reaches to zero, the third quarter takes control of the maneuver. Then, the other half of the loop is completed with $Loop3^{rd}Guarter$ state followed by $Loop4^{th}Guarter$ state.

3.4. Break Turn

A Break Turn is recommended to change direction as quick as possible. It is often used to avoid missiles or to outmaneuver enemy aircraft. The hybrid dynamics of the Break Turn as an automaton is illustrated in Fig. 5.

$Q = \{LevelFlight, Acceleration, Turn, Roll, LevelFlight\}$, the set of differential equations and other information are represented in Fig. 5. The input variable σ is used to initialize the maneuver. The other input parameters are $\dot{\mu}_{limit}$ and δ_{stable}. $\mu_{desired}$ is standardized as $0.89\pi/2$ for Break Turn. According to sign of $\dot{\mu}_{limit}$, turn direction is defined.

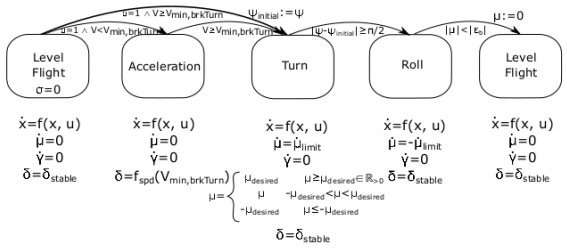

Figure 5. Hybrid Dynamics of Break Turn Maneuver.

In the Break Turn maneuver, the $Turn$ state corresponds to setting the bank angle to a desired value until 90° heading change. Then, the $Roll$ state is activated that changes the bank angle with constant bank angle rate until it reaches to zero.

3.5. Immelmann

An Immelmann is used to quickly obtain a reverse heading and bleed off excess speed or gain altitude.

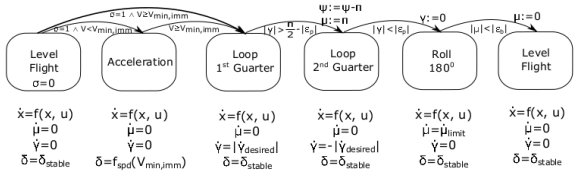

Figure 6. Hybrid Dynamics of Immelmann Maneuver.

In Fig. 6, the hybrid automaton of the Immelmann is illustrated with $Q, E, X, Init, Flow, Inv, G, R$ parameters. The input variables are σ, $\dot{\gamma}_{desired}$, $\dot{\mu}_{limit}$ and δ_{stable}. The expressions $\psi := \psi - \pi$ and $\mu = \pi$ are used as reset relations to obtain reverse heading.

Let us note that all of the discrete states in the Immelman were already presented in the previous maneuvers. However, the Immelman automaton uses these discrete states with a different sequence to implement Immelman maneuver.

3.6. Split S

A Split S is the opposite of an Immelmann. While it quickly makes 180^o turn as Immelmann, it sacrifices altitude for speed. It can be used to quickly lose altitude while making a 180^o turn. Because of this similarity, all of the discrete states in the hybrid automaton of the Split S are same with the states in the Immelman. But the sequences of the discrete states are different.

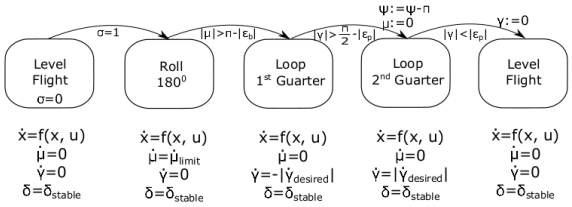

Figure 7. Hybrid Dynamics of Split S Maneuver.

In Fig. 7, the hybrid automaton of the Split S is illustrated with $Q, E, X, Init, Flow, Inv, G, R$ parameters. The input variables are σ, $\dot{\gamma}_{desired}$, $\dot{\mu}_{limit}$ and δ_{stable}.

3.7. Vertical Spiral

Most of the time a Vertical Spiral is a defensive maneuver. The maneuver consists of rising the nose up during the turn and going in to a spiral. The defender's goal is stay out of phase with the attacker. When the defender's aircraft has superiority in terms of climb and hard turn, then it will create an advantage.

The set of differential equations and other information about this hybrid automaton are represented in Fig. 8. The input variable σ is used to initialize the maneuver. The other input parameters are $\dot{\mu}_{limit}$, $\dot{\gamma}_{limit}$, $\mu_{desired}$, $\gamma_{desired}$, δ_{stable} and $\Delta h_{desired}$. $\Delta h_{desired}$ denotes the desired altitude change during maneuver.

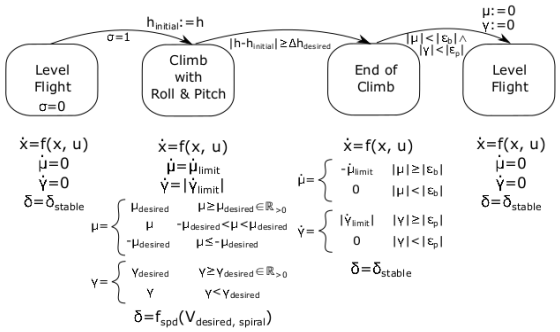

Figure 8. Hybrid Dynamics of Vertical Spiral Maneuver.

In the Vertical Spiral, the $Climb\ with\ Roll\&Pitch$ state contains climbing with a specific flight path angle, while turning with a fixed bank angle. The state is terminated, when the altitude change reaches the desired value. Then, the $End\ of\ Climb$ state tries to set the bank angle and flight path angle to zero via the bank angle rate and path angle rate.

3.8. Spiral Dive

Most of the time a Spiral Dive is a defensive maneuver. It is used by the defender when the kinetic energy become depleted. The maneuver consists of dropping the nose low

during the turn and going in to a spiral. The defender's goal is stay out of phase with the attacker. The hybrid automaton of the Spiral Dive is presented in Fig. 9.

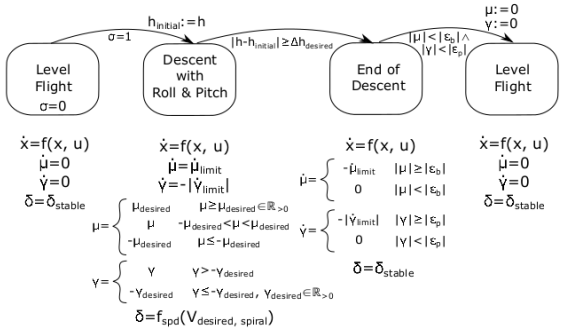

Figure 9. Hybrid Dynamics of Spiral Dive Maneuver.

In the Spiral Dive, the *Descent with Roll&Pitch* state works like the *Climb with Roll&Pitch* state in the Vertical Spiral. However, the Spiral Dive automaton uses negative path angle and path angle rate to generate maneuver.

3.9. Low Yo-Yo

Most of the time a Low Yo-Yo is an offensive maneuver. This maneuver is accomplished by rolling with the nose down into the turn and climbing back to the defender's height. It is an out-of-plane maneuver. It sacrifices altitude for an instantaneous increase in speed. The Low Yo-Yo is often followed by a High Yo-Yo, to help prevent an overshoot, or several small Low Yo-Yo's can be used instead of one large maneuver.

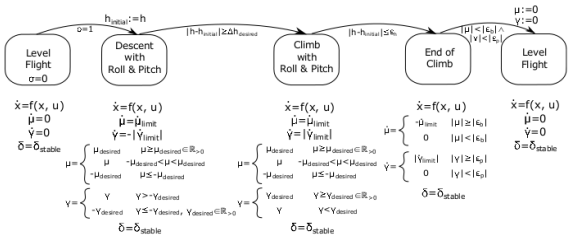

Figure 10. Hybrid Dynamics of Low Yo-Yo Maneuver.

In Fig. 10, the hybrid automaton of the Low Yo-Yo is portrayed with $Q, E, X, Init, Flow, Inv, G, R$ parameters. The input variables are σ, $\dot{\mu}_{limit}$,$\dot{\gamma}_{limit}$, $\mu_{desired}$, $\gamma_{desired}$, δ_{stable} and $\Delta h_{desired}$. $\Delta h_{desired}$ corresponds to the desired altitude change during maneuver. ϵ_h is the error tolerance for altitude and it is defined as $5m$.

The Low Yo-Yo automaton uses the discrete states that are presented in the Vertical Spiral and Spiral Dive automaton. This automaton uses the *Descent with Roll&Pitch* state followed by the *Climb with Roll&Pitch* state for rolling with the nose down into the turn and climbing back to the previous altitude.

3.10. High Yo-Yo

Most of the time a High Yo-Yo is an offensive maneuver. This maneuver is accomplished by rolling with the nose up into the turn and diving back to the defender's height. It is an out-of-plane maneuver. The maneuver is used to slow the approach of a fast moving attacker while conserving the airspeed energy.

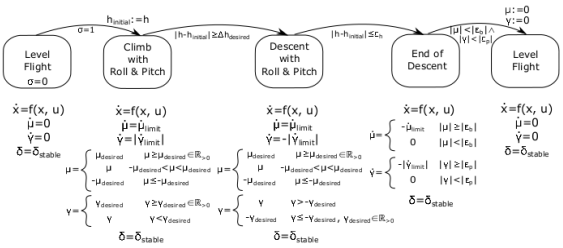

Figure 11. Hybrid Dynamics of High Yo-Yo Maneuver.

In Fig. 11, the hybrid automaton of the High Yo-Yo is portrayed with $Q, E, X, Init, Flow, Inv, G, R$ parameters. The automaton uses the same discrete states in the Low Yo-Yo in a different order. Whereas the aircraft firstly descends and then climbs in the Low Yo-Yo, it firstly climbs and then dives in the High Yo-Yo. The only difference is the sequences of the states. The Low Yo-Yo and High Yo-Yo are illustrated in Figure 1.i and Figure 1.j, respectively.

4. QUANTIFICATION OF COMBATANT'S OBJECTIVES

The relative position, angular relationship, speed and altitude are main factors that affect the air superiority [12],[13]. The objectives of the combatants can be quantified as a function of these variables. This section contains the explanations of the objective functions.

4.1. Offensive Orientation Score

As shown in Fig. 12, the aspect angle of the blue aircraft AA_b describes the angle from the tail of red aircraft to the position of the blue aircraft. The bearing angle of the blue combatant BA_b denotes the angle from the head of blue to the position of the red aircraft. The range is the distance between two aircraft.

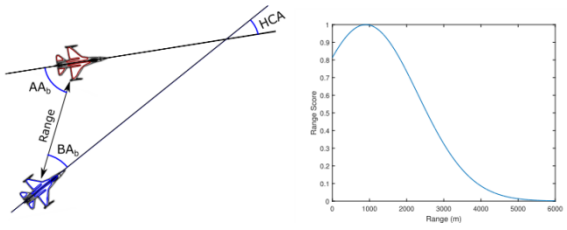

Figure 12. Combat Geometry and Range Score Function.

In offensive manner, each aircraft tries to locate at the rear of the other aircraft with same heading angle for air superiority. When AA_b and BA_b are zero, blue's orientation is perfect due to the alignment with red and also from behind. When AA_b and BA_b are π, it is the worst case for blue. The sum of AA_b and BA_b should be minimized for air superiority of blue. Then, the offensive orientation score for blue is described by the following relationship:

$$S_{o_1} = 1 - \frac{AA_b + BA_b}{\pi} \tag{12}$$

This orientation score should be maximized for the blue aircraft. The score takes value between -1 and 1. Blue is on red's tail when score is 1, and red is on blue's tail when it is -1.

4.2. Defensive Orientation Score

In defensive manner, an aircraft tries to escape from dangerous orientations that correspond to both aspect and bearing angle of opponent are smaller than 30^o. The defensive orientation score for blue is described as follows:

$$S_{o_2} = \begin{cases} 1 & if\ AA_r > \frac{\pi}{6}\ or\ BA_r > \frac{\pi}{6} \\ 0 & otherwise \end{cases} \tag{13}$$

4.3. Range Score

The other scoring function is the range score. During combat, the fighter tries to shoot down the other one. For shooting, a safety distance is necessary and this distance also must be smaller than the range of the missile or gun. To define desired range interval, following range score is used.

$$S_{rg} = a_1 e^{\frac{-(R-R_d)^2}{2\sigma^2}} \tag{14}$$

which is a gaussian distribution multiplied by the normalization parameter a_1. $R \in \mathbb{R}_{>0}$ is the range between two aircraft. R_d is the desired range and σ is the standard deviation. The shooting range for a short range air-to-air missile can be from 900m to 5km. $R_d = 900$ and $\sigma = 1400$ are chosen to construct these range limitations. The illustration of the range score with these parameters is given in Fig. 12.

4.4. Speed Score

The total energy level of an aircraft is crucial in an air combat. So, the speed of the aircraft should be held at desired level to prevent the high amount of speed decrease. The scoring function for the speed is given as follows:

$$S_v = \begin{cases} a_2 & If\ \left|\frac{V_{cur} - V_{ref}}{V_{ref}}\right| > 0.5 \\ a_2 \left|\frac{V_{cur} - V_{ref}}{V_{ref}}\right| & otherwise \end{cases} \tag{15}$$

where, $a_2 \in \mathbb{R}_{<0}$ is the scale parameter and is chosen as -0.8 in this study. V_{cur} and V_{ref} are the current speed and reference/desired speed, respectively.

4.5. Altitude Score

The other factor is the altitude. Following scoring function is used to prevent the undesirable altitude decrease.

$$S_h = \begin{cases} a_3 & If\ |H_{cur} - H_{ref}| \le 2000 \\ 0 & otherwise \end{cases} \tag{16}$$

where, $a_3 \in \mathbb{R}_{>0}$ is the score parameter and is chosen as 0.5 in this study. H_{cur} and H_{ref} are the current altitude and reference/desired altitude, respectively.

4.6. Total Scores for 2vs1 Air Combats

The objective function is the combination of these scoring functions. In $2vs1$ combat, the blue team has 2 combatants, while the red team has only 1. However, team members have cooperation. So, only one objective function is developed for each team. The general combined objective function is at the form of (17).

$$S_t = S_o S_{rg} + S_v + S_h \tag{17}$$

The objective is to maximization of S_t. S_{o_1} is used as S_o in offensive mode, while S_{o_2} is used as S_o in defensive mode. When both fighters are offensive against enemy, the total scoring function for the blue team in $2vs1$ combat is given by the following equation.

$$S_{b,t} = S_{o_1,b_1r_1} S_{rg,b_1r_1} + S_{o_1,b_2r_1} S_{rg,b_2r_1} + S_V + S_H \tag{18}$$

where, b_1, b_2 and r_1 correspond to 1^{st} fighter in the blue team, 2^{nd} fighter in the blue team and fighter in the red team, respectively. $S_V = S_{v,b_1} + S_{v,b_2} - S_{v,r_1}$ is the combined speed score of b_1, b_2 and r_1. $S_H = S_{h,b_1} + S_{h,b_2} - S_{h,r_1}$ is the combined altitude score.

The objective function is defined according to strategy against other team. Let us consider that b_1 is offensive, while b_2 is defensive against r_1, then the objective function is given as follows:

$$S_{b,t} = S_{o_1,b_1r_1}S_{rg,b_1r_1} + S_{o_2,b_2r_1}S_{rg,b_2r_1} + S_V + S_H \quad (19)$$

In this scoring function, the defensive orientation score S_{o_2} is used instead of the offensive orientation S_{o_1} for b_2.

The objective function for the red team is also determined with same approach. When r_1 is defensive against b_1 and r_1 is offensive against b_2, the total scoring function is given as follows:

$$S_{r,t} = S_{o_2,r_1b_1}S_{rg,r_1b_1} + S_{o_1,r_1b_2}S_{rg,r_1b_2} - S_V - S_H \quad (20)$$

In this concept, it can be said that aggressiveness of the fighter determines the total score function. Aggressiveness of a combatant against to an enemy can be low (defensive) or high (offensive). It can be presented as boolean. Defensive manner corresponds to 0, while offensive manner corresponds to 1. Therefore, the combination of aggressiveness in a $2vs1$ combat can be presented with a 4-digit binary number. E.g. let us b_1 and b_2 be offensive against to r_1, r_1 be defensive against to b_1 and offensive against to b_2, then the combination of aggressiveness is presented as 1101. This presentation will be used at the rest of the paper.

5. SECURITY STRATEGY TO EVALUATE THE NONZERO-SUM GAME

A two person nonzero-sum game [3], [6] is given by a collection (X, Y, A, B), where

- X is a set of the strategies of Player 1 $(P1)$
- Y is a set of the strategies of Player 2 $(P2)$
- A is payoff function of $P1$ defined on $X \times Y$
- B is payoff function of $P2$ defined on $X \times Y$

In this game, simultaneously, $P1$ chooses $x \in X$ and $P2$ chooses $y \in Y$, the choice of the other is unknown for each. In this finite two person nonzero-sum game or bimatrix game, if $X = \{x_1, \dots, x_m\}$ and $Y = \{y_1, \dots, y_n\}$, then the game matrices are given as A, B and $a_{ij} = A(x_i, y_j)$, $b_{ij} = B(x_i, y_j)$ are the gains, when $P1$ chooses row i and $P2$ chooses column j, respectively.

As in the zero-sum games, security strategies can be introduced for the players in the bimatrix games. The security strategy for a player is presented such that the player's average gain is at least V no matter what the other player does. The strategy creates a lower bound for the gain of the player that is independent from the choice of the other player. It should be emphasized that, in a bimatrix game (A, B), the security strategies of $P1$ involve only the entries of matrix A, while the entries of matrix B are used to generate the security strategies of $P2$. Let row i^* be the security strategy for $P1$ in the matrix game A and column j^* be the security strategy for $P2$ in the matrix game B, then the pair of strategies (i^*, j^*) is known as a pair of minimax strategies and also a pure strategies for the players in a bimatrix game (A, B).

A mixed strategy for $P1$ can be denoted by an m-tuple, $p = (p_1, p_2, \dots, p_m)^T$ of probabilities that add to 1. Similarly, a mixed strategy for $P2$ is represented as an n-tuple, $q = (q_1, q_2, \dots, q_n)^T$.

Let us focus on the problem from perspective of $P1$. So, let us use the payoff matrix A. Suppose that $P2$ selects a column at random using $q \in Y^*$. If $P1$ prefers row i, the $P1$'s average payoff is calculated as

$$\sum_{j=1}^{n} a_{ij}q_j = (Aq)_i \quad (21)$$

where, $(Aq)_i$ corresponds to the i^{th} component of the vector Aq. Similarly, if $P1$ chooses $p \in X^*$ and $P2$ prefers column j, then the average payoff to $P1$ becomes

$$\sum_{i=1}^{m} p_i a_{ij} = (p^T A)_j \quad (22)$$

where, $(p^T A)_j$ denotes the j^{th} component of the vector $p^T A$. More generally, if $P1$ uses $p \in X^*$ and $P2$ chooses $q \in Y^*$, the average payoff to $P1$ is

$$\sum_{i=1}^{m}(\sum_{j=1}^{n} a_{ij}q_j)p_i = p^T Aq \quad (23)$$

Suppose it is known that Player 2 is going to use a particular strategy $q \in Y^*$. Then $P1$ would prefer that row i that maximizes (21); or, equivalently, he would choose that $p \in X^*$ that maximizes (23). The average payoff to $P1$ would be

$$\max_{1 \le i \le m} \sum_{j=1}^{n} a_{ij}q_j = \max_{p \in X^*} p^T Aq \quad (24)$$

To see that these quantities are equal, note that the left side is the maximum of $p^T Aq$ over $p \in X^*$, and so, since $X \subseteq X^*$, must be less than or equal to the right side.

The solution $p \in X^*$ of the equality (24) is called a best response or a Bayes strategy against q. In particular, when the maximum of (21) is achieved by row i, this strategy i is called as a pure Bayes strategy against q.

Similarly, if the preferred strategy $p \in X^*$ of $P1$ is known, then $P2$ would choose that column j that minimizes (22), or, equivalently, that $q \in Y^*$ that minimizes (23). The average loss for $P1$ would be

$$\min_{1 \le j \le n} \sum_{i=1}^{m} p_i a_{ij} = \min_{q \in Y^*} p^T Aq \quad (25)$$

Any $q \in Y^*$ that achieves the minimum in (25) is called a best response or a Bayes strategy for $P2$ against p. The notion of a best response presents a practical way of playing a game: Make a guess at the probabilities that you think your opponent will play his/her various pure strategies, and choose a best response against this.

The maximum of (25) over all $p \in X^*$ is denoted by $\underline{V}$ and called $the\ lower\ value\ of\ the\ game\ (X, Y, A)$.

$$\underline{V} = \max_{p \in X^*} \min_{1 \le j \le n} \sum_{i=1}^{m} p_i a_{ij} = \max_{p \in X^*} \min_{q \in Y^*} p^T A q \quad (26)$$

It is the maximum amount that $P1$ can guarantee himself no matter what $P2$ does. Any $p \in X^*$ that achieves the maximum in (26) is called a minimax strategy for $P1$.

For the solution [6] of minimax strategies, linear programming can be used. Let us consider the game problem from $P1$'s point of view. He wants to choose $p = (p_1, p_2, \ldots, p_m)^T$ to maximize (25) subject to the constraint $p \in X^*$. This becomes the mathematical program: choose $p = (p_1, p_{,2}, \ldots, p_m)^T$

$$maximize \min_{i \le j \le n} \sum_{i=1}^{m} p_i a_{ij} \quad (27)$$

subject to:

$$p_1 + \ldots + p_m = 1 \quad (28)$$

$$p_i \ge 0 \quad for\ i = 1, \ldots, m \quad (29)$$

Because of the min operator on objective function, this is not a linear programming. However, it can be changed through a trick. Variable v is added to the list of $P1$'s variables and it is restricted to be less then objective function, $v \le min \sum_{i=1}^{m} p_i a_{ij}$, and it is tried to make v as large as possible. The problem becomes:

Choose v and $p = (p_1, p_2, \ldots, p_m)^T$ to

$$maximize \quad v \quad (30)$$

subject to:

$$v \le \sum_{i=1}^{m} p_i a_{i1}$$

$$\vdots \quad (31)$$

$$v \le \sum_{i=1}^{m} p_i a_{in}$$

$$p_1 + \ldots + p_m = 1 \quad (32)$$

$$p_i \ge 0 \quad for\ i = 1, \ldots, m \quad (33)$$

This is a standard linear programming. The solution of this linear programming gives the mixed strategies of $P1$ as $p = (p_1, \ldots, p_m)^T$. Same procedure can be applied for $P2$ to generate the mixed strategy $q = (q_1, \ldots, q_n)^T$ by using payoff matrix B instead of matrix A.

In $2vs1$ air combat, when fighters are engaged two different 100×10 payoff matrices are generated that correspond to A and B according to score functions $S_{b,t}$ and $S_{r,t}$, respectively. The score functions are determined according to predefined 4-digit binary number, which presents the combination of aggressiveness for the game. Each aircraft has 10 maneuver choices. Therefore, the blue team has 100 maneuver combinations, while the red team has 10 maneuver choices. These maneuvers are compared one by one, and an average score is calculated for each maneuver combination to generate payoff matrices. After payoff matrices are calculated, linear programs are used to generate the probability vectors $p = (p_1, \ldots, p_m)^T$ for the blue team and $q = q_1, \ldots, q_n)^T$ for the red team. These two vectors represent the probabilities to choose a maneuver combination for each team with optimal strategy. There are two possibilities in the simulation environment, the simulation can be executed in stochastic mode according to probability vectors or deterministic mode that uses the maneuver which has maximum probability in probability vector for each team. After a team completes the chosen maneuvers, then the process is repeated by considering the trajectory of the other aircraft that stays from the previous choice. The simulation finishes when a team shoots down the other or predefined simulation time ends.

6. IMPLEMENTATIONS AND RESULTS

A specific set of input parameters is required to simulate a maneuver in air combat maneuver library that determines the movement space of the maneuver. In the implementation phase, input parameters for maneuvers are standardized as given in Table 1. The input parameters are defined heuristically for implementation by considering the real situation, performance limits of the fighters and synchronization of the maneuvers both in time and space. As an alternative approach, a dynamical selection method can be used that evaluates the energy levels and current situations of the fighters to determine the input parameters of the maneuvers. However, in this study, we focus on a specific set of parameters to simulate and validate the proposed methodology.

Table 1. Input Parameters for Simulations.

	μ_d °	γ_d °	$\dot{\mu}_d$ °/s	$\dot{\gamma}_d$ °/s	$\dot{\mu}_l$ °/s	$\dot{\gamma}_l$ °/s	Δh_d m
Aileron R.	-	-	40	-	-	-	-
Barrel R.	-	30	40	-	-	90	-
Loop	-	-	-	40	-	-	-
Break T.	80	-	-	-	90	-	-
Immelma.	-	-	-	40	90	-	-
Split S	-	-	-	40	90	-	-
Vertical Sp.	80	30	-	-	90	90	500
Spiral D.	80	30	-	-	90	90	500
Low Y-Y	65	30	-	-	90	90	500
High Y-Y	65	30	-	-	90	90	500

Three different scenarios are analyzed in deterministic mode. These scenarios are two-on-one air combats at different combination of aggressiveness. The initial conditions for the fighters are presented in Table 2. The combination of aggressiveness for scenarios are 1111, 1101 and 1001, respectively. Besides these main scenarios, an additional scenario is evaluated to observe the impact of different defensive orientation functions on the results.

Table 2. Initial Conditions for Scenarios.

	x_1	x_2	h	V	ψ
Blue 1	10000	10000	6000	250	$\pi/3$
Blue 2	14000	10000	6000	250	$2\pi/3$
Red 1	12000	10000	6000	250	$\pi/2$

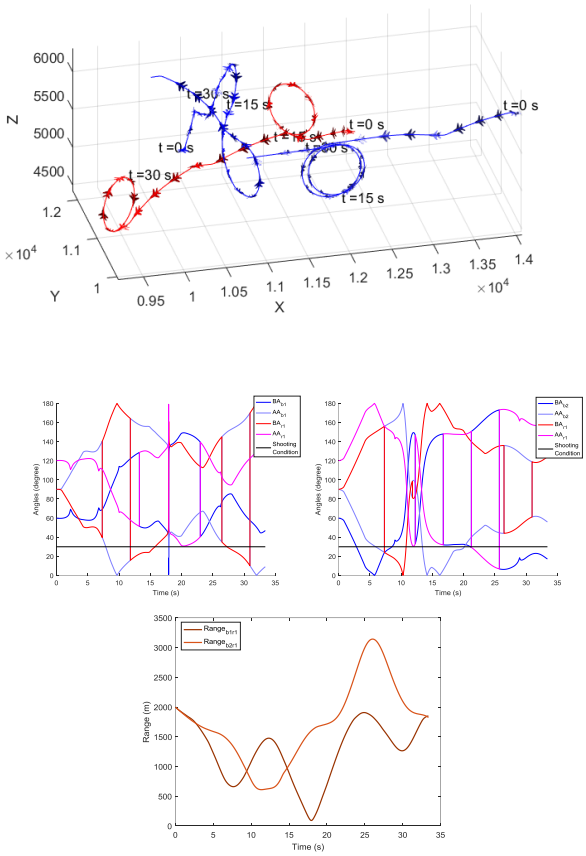

Figure 13. Air Combat for 1111 Configuration.

In all of the main scenarios, initial conditions except aggressiveness are same. In the first scenario, all of the fighters are offensive against the other enemies. We assume that minimum $3000\,m$ is necessary to shoot down and both aspect angle and bearing angle of the attacker must be smaller that 30^o for a shooting. The simulation results are represented in Fig. 13. Blue 1 chooses the sequence of maneuvers as Barrel Roll, Split S, Spiral Dive, Immelmann, Vertical Spiral, while Blue 2 chooses the sequence as Spiral Dive, Break Turn, Split S, Loop, Immelmann, Break Turn. The sequence of maneuvers is Spiral Dive, Loop, Spiral Dive, Spiral Dive, Loop and Vertical Spiral for the red aircraft. The bearing and aspect angles of the teams are illustrated in the Fig. 13b and Fig. 13c and the ranges between aircraft are also presented in Fig. 13d. During air combat, there is no air superiority between Blue 1 and Red, while Blue 2 has $1.39s$ air superiority, which begins at $6s$ of the simulation. During this superiority period, the ranges between Blue 2 and Red are around $1500m$ that is sufficient for a shooting.

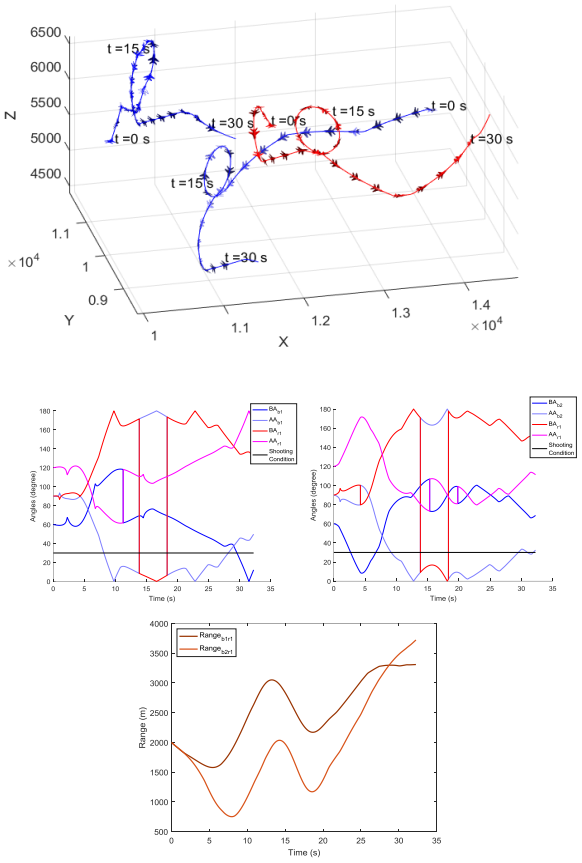

Figure 14. Air Combat for 1101 Configuration.

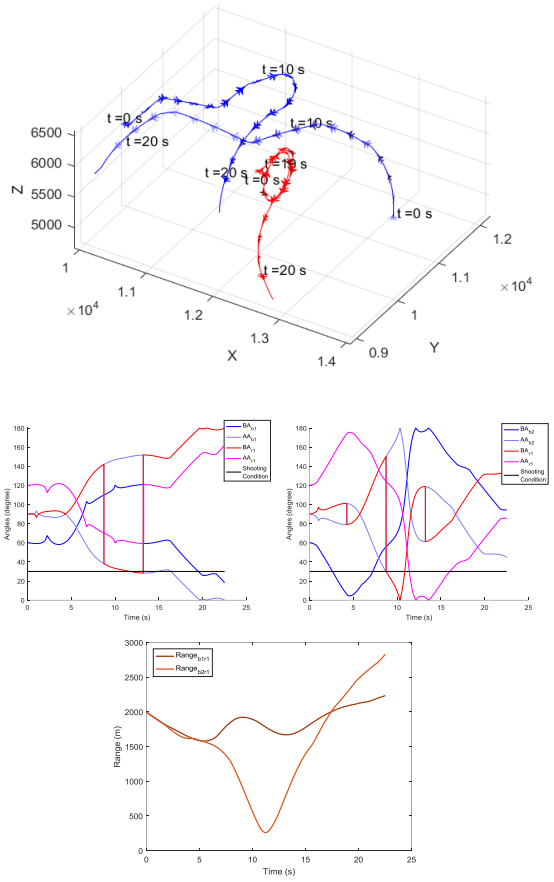

Figure 15. Air Combat for 1001 Configuration.

In the second scenario, Red is defensive against Blue 1, while it is offensive against Blue 2. And, both members of the blue team are offensive. The simulation results are illustrated in Fig. 14. The sequence of maneuvers is Barrel Roll, Immelmann, Low Yo-Yo, Spiral Dive for Blue 1 and it is Spiral Dive, Break Turn, Spiral Dive, Loop, Spiral Dive, Vertical Spiral for Blue 2. The maneuver set is Split S, Vertical Spiral, Loop, Spiral Dive, Vertical Spiral, Vertical Spiral for Red. During the combat period, none of the combatants has a chance for a shooting because of insufficient orientations as seen in Fig. 14b and Fig. 14c. Therefore, this aggressiveness strategy is better than 1111 from perspective of the red team, whereas 1111 is better for the blue team.

In the third scenario, Blue 1 and Red are offensive against Red and Blue 2, whereas Blue 2 and Red are defensive against Red and Blue 1, respectively. The simulation results are illustrated in Fig. 15. Blue 1 chooses the sequence of maneuvers as Barrel Roll, Split S, Spiral Dive, Spiral Dive, whereas Blue 2 chooses the sequence as Vertical Spiral, Break Turn, Low Yo-Yo and Spiral Dive. The maneuver set is Split S, Loop, Spiral Dive and Aileron Roll for Red aircraft. As seen in Fig. 15b, Blue 1 has air superiority against Red during $3s$, then the game is terminated. While 1101 is better for the red team, 1001 is best for the blue team. Changing the aggressiveness of Blue 2 from 1101 to 1001, the blue team can get the advantage. This strategy is totally related to the blue team. Changing only one of the combatant's mood, air combat can be won. In the 1111 combination, the blue team has air superiority for $1.39s$ time period. Changing the mood of Red from 1111 to 1101, the air combat can be ended in a draw, which is more desirable for the red team.

In the additional scenario, the aggressiveness is 1001 as in the third scenario. However, the threshold of the defensive orientation score is defined as $\pi/3$ instead of $\pi/6$ in the equation (13). The simulation results are represented in Fig. 16. As shown in the results, the blue team chooses the same maneuvers in the third scenario. The red aircraft also selects the same maneuvers at the beginning of the combat. However, it acts more agile then the previous scenario after second maneuver. It chooses the Break Turn instead of the Spiral Dive as third maneuver due to the new defensive orientation score. But, this choice can not change the outcome. Blue 1 still has air superiority against Red during $3s$, then the game is terminated.

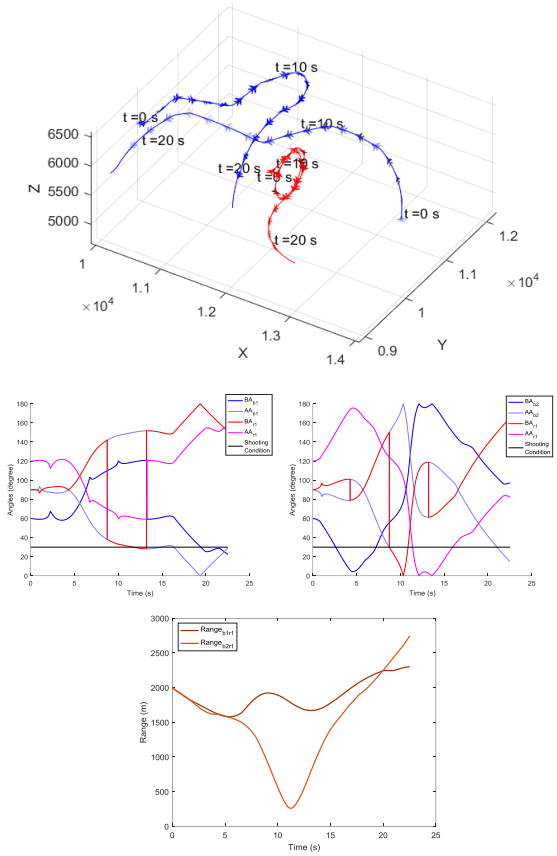

Figure 16. Air Combat for 1001 Configuration with Different Defensive Score Function.

7. CONCLUSION

In this paper, a simulation platform and maneuver selection methodology for two-vs-one air combats were presented. We used an aircraft model for F-16 to implement the movements of the fighters. Known air combat maneuvers were formulated through hybrid automaton spanning maneuver space of the aircraft. A typical objective function for air superiority was given and applied. Then, game-theoretical evaluation strategy was expressed. The working principles of the method were presented and the validity of the algorithm was demonstrated through implementations.

8. ACKNOWLEDGEMENTS

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